NUCLEAR RESPONSE TO MULTI-GEV NEUTRINOS IN THE RELATIVISTIC MEAN FIELD MODEL

- Description of the model and hypothesis
- Consequences of the model: no (exact) factorization but still (super)scaling
- Validation of the model against data: Inclusive QE electron scattering
- Applicability of the model to neutrinos

In collaboration with many people whose work I quote liberally:

E. Amaro, A. Antonov, M. Barbaro, J.A. Caballero, T.W. Donnelly, M. Gaidarov, M. Ivanov, P. Lava, C. Maieron, E. Moya, M. C. Martínez, Jan Ryckebusch et al., Javier R. Vignote...

What do we mean by Relativistic Mean Field (RMF) Model?

- We mean the use of the Dirac equation with its relativistic treatment of dynamics and kinematics as opposed to the nonrelativistic Schrödinger equation (which can also include relativistic kinematics) to describe single nucleon motion in nuclei

- Relativity is important at low energies, and even at zero incident energy!!!

- The Dirac equation provides a natural description of spin-1/2 particles and, hence, provides a good framework for studying spin observables

Scalar *and* Vector potentials

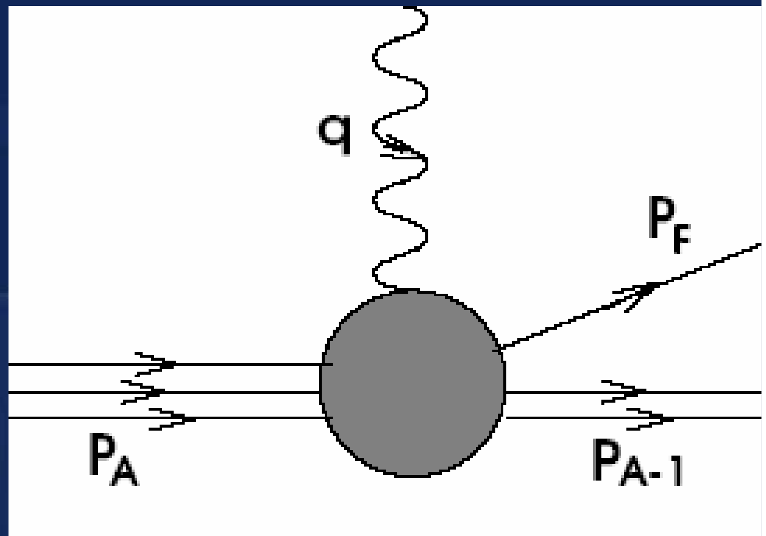
$$(\tilde{E}\gamma_0 - \vec{p}\cdot\vec{\gamma} - \tilde{M})\psi = 0$$

$$\tilde{E} = E - V(r)$$
$$\tilde{M} = M - S(r)$$

- Solve a Dirac-like equation
- Bound state: Phenomenological σ-ω lagrangeans (Serot and Walecka model and extensions) at mean field level adjusted to reproduce binding energy and radii of some doubly magic nuclei or nuclear saturation properties
- Final State: Optical Potentials (exclusive scattering) or the same mean field S-V potentials as for bound states (inclusive scattering)
- **DW** approach, describing the final state by means of partial waves

OVERVIEW OF THE MODEL (ingredients)

1) Weak interacting probe (e^-,ν_e ...). It allows for the simplest approach: single boson (photon, $W^\pm$,Z^0) exchange



2) Thus, the dependence on the kinematics of the exchanged boson can be extracted. For unpolarized and in plane electron scattering, this means:

$$\frac{d\sigma}{d\Omega_e d\varepsilon' d\Omega_F} = K\sigma_{Mott} f_{rec}\left[v_L R^L + v_T R^T + v_{TL} R^{TL}\cos\phi_F + v_{TT} R^{TT}\cos 2\phi_F\right]$$

One-boson exchange approximation yields, for the most general case:

$$\frac{d\sigma}{d\varepsilon_f d\Omega_f d\Omega_F} = \frac{E_F p_F}{(2\pi)^3}\sigma_M f_{rec}\frac{1}{2}\left\{v_L\left(R^L + R^L_n\widehat{S}_n\right) + v_T\left(R^T + R^T_n\widehat{S}_n\right)\right.$$
$$+ v_{TL}\left[\left(R^{TL} + R^{TL}_n\widehat{S}_n\right)\cos\phi_F + \left(R^{TL}_l\widehat{S}_l + R^{TL}_s\widehat{S}_s\right)\sin\phi_F\right]$$
$$+ v_{TT}\left[\left(R^{TT} + R^{TT}_n\widehat{S}_n\right)\cos 2\phi_F + \left(R^{TT}_l\widehat{S}_l + R^{TT}_s\widehat{S}_s\right)\sin 2\phi_F\right]$$
$$+ h\left\{v_{TL'}\left[\left(R^{TL'}_l\widehat{S}_l + R^{TL'}_s\widehat{S}_s\right)\cos\phi_F + \left(R^{TL'} + R^{TL'}_n\widehat{S}_n\right)\sin\phi_F\right]\right.$$
$$\left.\left. + v_{T'}\left[R^{T'}_l\widehat{S}_l + R^{T'}_s\widehat{S}_s\right]\right\}\right\},$$

R (response) functions are proportional to the Hadronic tensor $W^{\mu\nu}$

$$W^{\mu\nu} = \frac{1}{2j_b+1}\sum_{\mu_b} J^{\mu *}(\omega, \mathbf{q}) J^{\nu}(\omega, \mathbf{q}) .$$

The one-boson exchange approximation allows us to decouple the direct dependence on the *energy* and *scattering angle* of the probe *via* the Mott cross-section for electrons or the equivalent expressions for neutrinos

NUCLEAR RESPONSE TO MULTI-GEV NEUTRINOS IN THE RELATIVISTIC MEAN FIELD MODEL

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$$\left(d\sigma^{Z^0/W^\pm}\right)_{\text{Free}} = \delta^{(4)}(k_i^\mu - k_f^\mu + P_I^\mu - P_F^\mu)\, \sigma^{Z^0/W^\pm} \frac{1}{4\epsilon_f^2 E_I E_F}\, \omega_{\mu\nu} W^{\mu\nu}\, d^3\vec{P}_F d^3\vec{k}_f$$

The hadronic part does not need to be computed at every point

$$\sigma^{Z^0} = 16\, \epsilon_f^2 \cos^2(\theta/2) \left[\frac{g^2}{4\pi}\right]^2$$

$$\sigma^{W^\pm} = 16 k_f^2 \left[\frac{g^2}{4\pi}\right]^2$$

$$\omega_L W_L = \frac{1}{4\epsilon_i k_f}\left\{ \left[(\epsilon_i + \epsilon_f)^2 - |\vec{k}\,|^2 - m_l^2\right] |\rho|^2 + \left[\frac{(\epsilon_i^2 - k_f^2)^2}{|\vec{k}\,|^2} - \omega^2 + m_l^2\right] |J_k|^2 - \left[\frac{2(\epsilon_i + e_f)(\epsilon_i^2 - k_f^2)}{|\vec{k}\,|} - 2\omega|\vec{k}\,|\right] Re\,(\rho^* J_k) \right\}$$

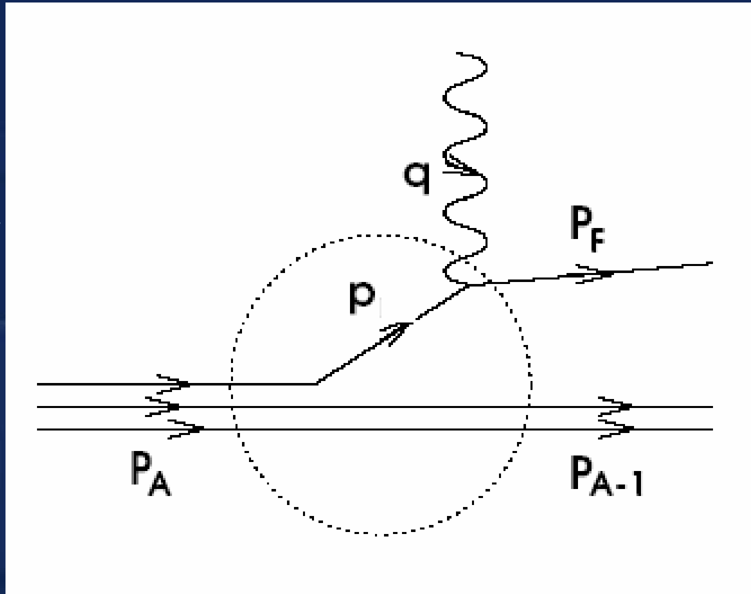
$$\omega_T W_T = \left\{ \cancel{\frac{\epsilon_i k_f \sin^2\theta}{2|\vec{k}\,|^2} \cos(2\phi_F)\left(|J_{||}|^2 - |J_\perp|^2\right)} + \left[\frac{\epsilon_i k_f \sin^2\theta}{2|\vec{k}\,|^2} - \frac{1}{2}\left(\frac{-\epsilon_f}{k_f} + \cos\theta\right)\right] \left(|J_{||}|^2 + |J_\perp|^2\right) \right\}$$

$$\omega_{TT'} W_{TT'} = -\frac{1}{|\vec{k}\,|}\left(\frac{\epsilon_i \epsilon_f}{k_f} + k_f - (\epsilon_i + \epsilon_f)\cos\theta\right) Im\,(J_{||} J_\perp^*)$$

L, T and TT' are the only responses that contribute if no nucleon is observed

OVERVIEW OF THE MODEL (more ingredients)

3) A further simplification: Impulse Approximation
A weak probe will interact with similar probability with both surface nucleons or deep ones



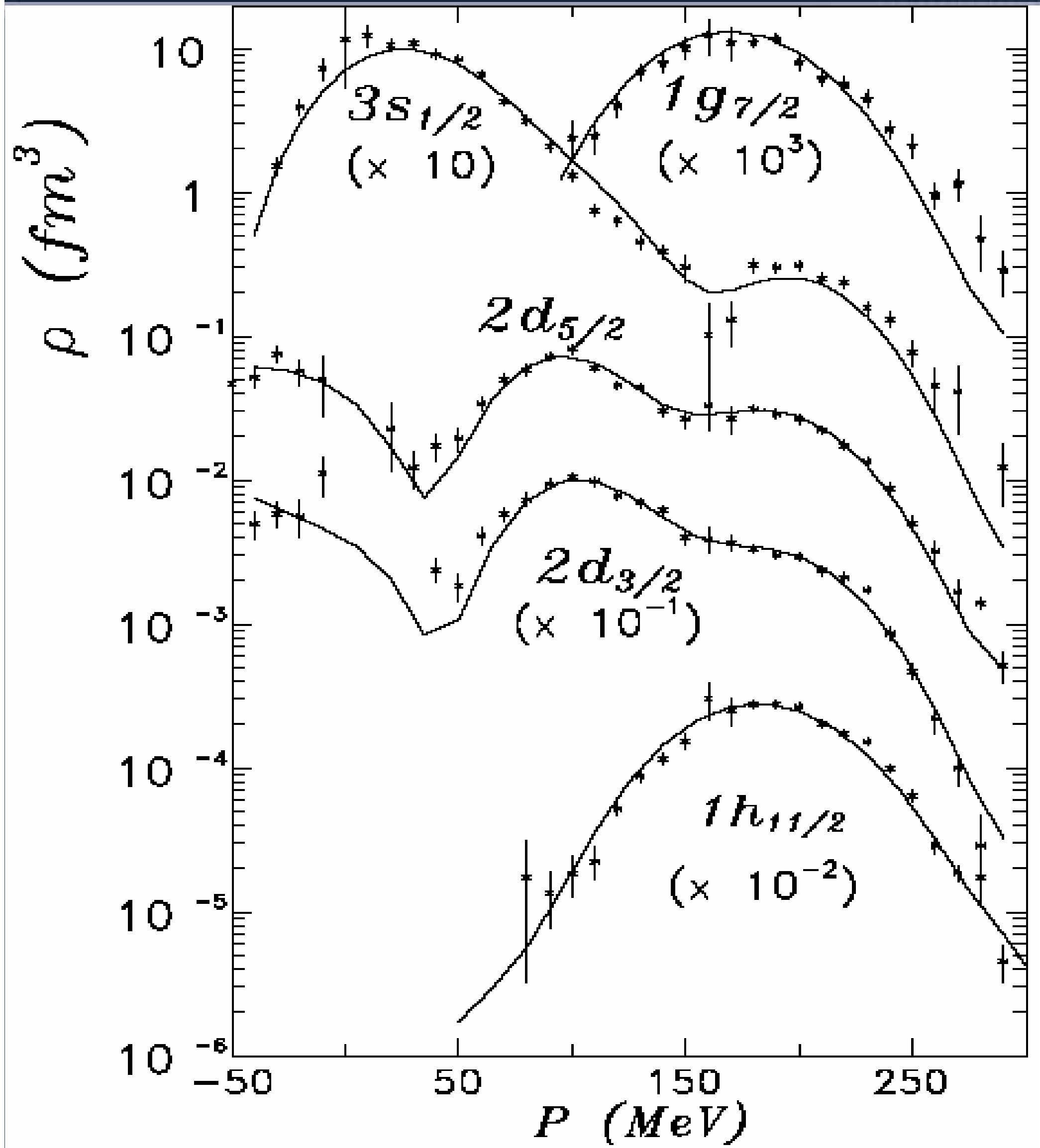
For QE conditions and large q (a few hundreds of MeV), all nucleons contribute to the cross-section incoherently. The nuclear current is obtained as a sum over individual single-nucleon currents:

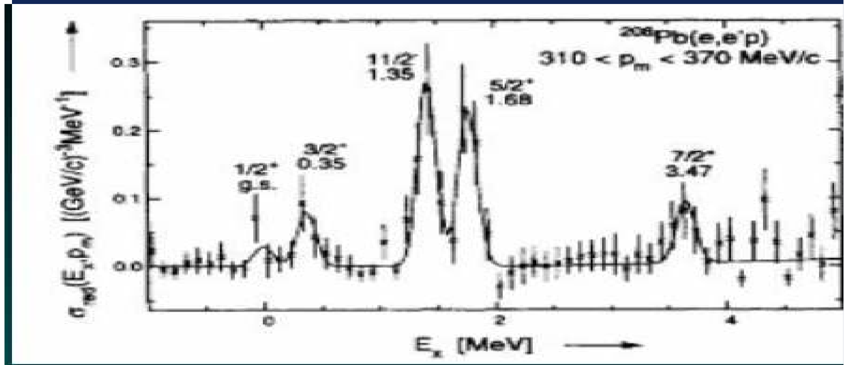
$$J_N^\mu(\omega,\vec{q}) = \int d\vec{p}\,\bar{\psi}_F(\vec{p}+\vec{q})\hat{J}_N^\mu(\omega,\vec{q})\psi_B(\vec{p})$$

SUMMARIZING SO FAR

- Under one boson exchange, direct dependence on energy and scattering angle is factored out
- Under Impulse Approximation, nuclear response is built from the sum of individual nucleon responses. Final phase space includes one nucleon knock-out factors
- Responses depend only on q and ω transferred to the nucleus, even when FSI or spinor distortions are considered

NUCLEAR RESPONSE TO MULTI-GEV NEUTRINOS IN THE RELATIVISTIC MEAN FIELD MODEL



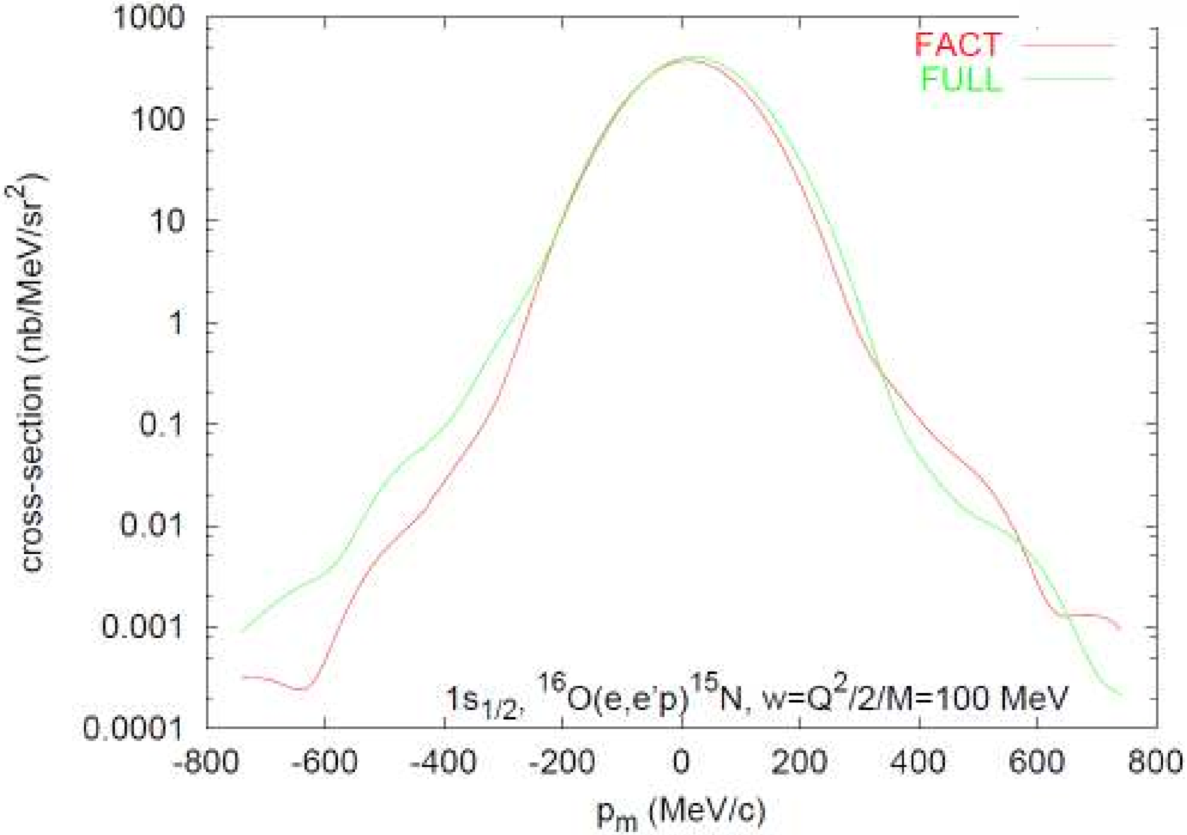


Reasonably good agreement with data in parallel kinematics

	$3s_{1/2}$	$2d_{3/2}$	$1h_{11/2}$	$2d_{5/2}$	$1g_{7/2}$
Non rel. (Ref. [41])	50 %	53 %	42 %	44 %	19 %
Non rel. (Ref. [42])	55 %	57 %	58 %	54 %	26 %
Rel. (Refs. [40, 6])	70 %	72 %	64 %	60 %	30 %

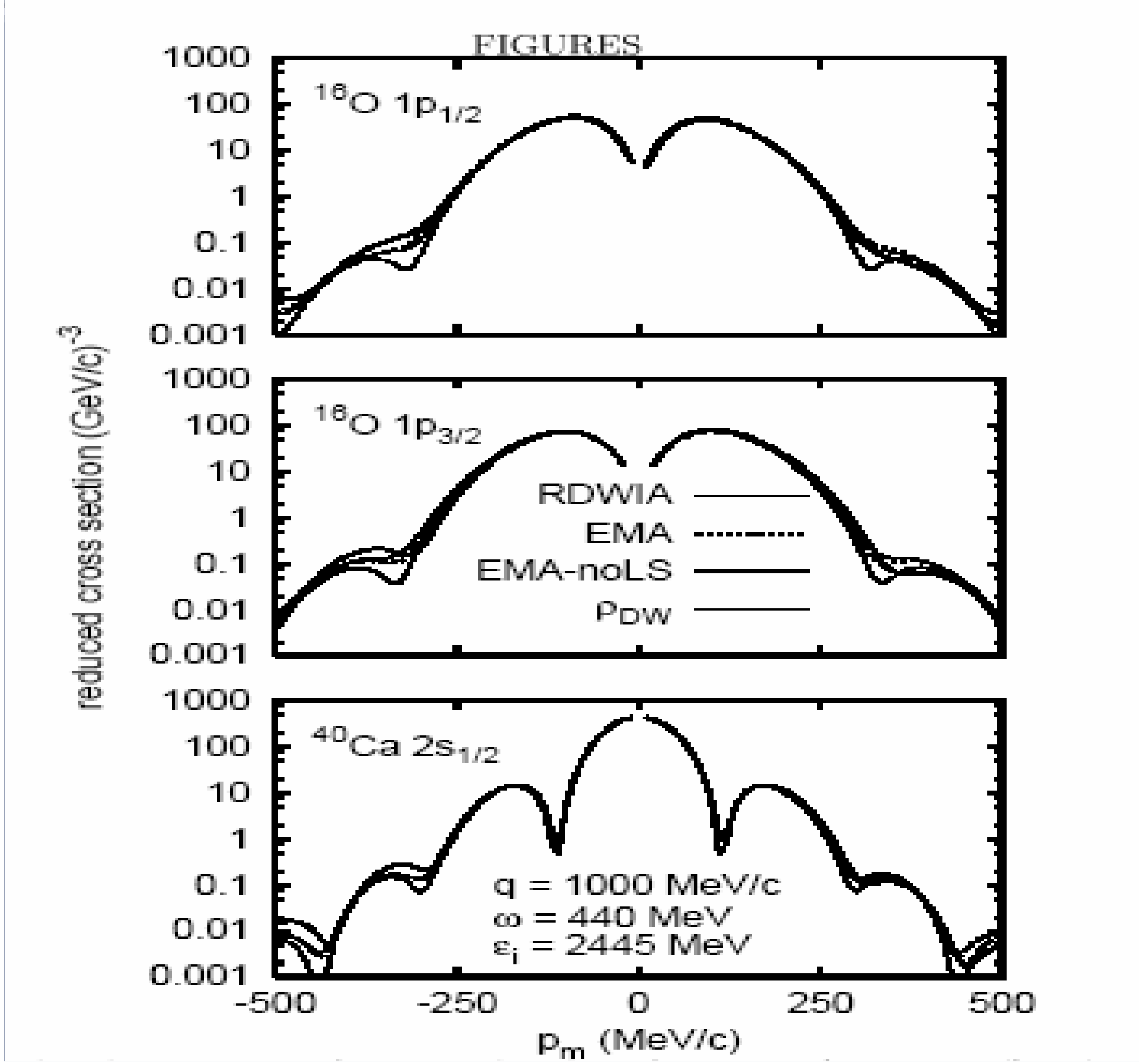
An useful tool in the interpretation of experiments: factorization approach

$$\frac{d^5\sigma}{d\Omega_e d\varepsilon' d\Omega_F} = K\sigma_{ep} S(E_m, \vec{p}_m) \qquad \rho^{exp}(\mathbf{p}_m) = \frac{\left(\frac{d\sigma}{d\varepsilon_f d\Omega_f d\Omega_F}\right)^{exp}}{E_F p_F f_{rec} \sigma_{ep}}\,.$$

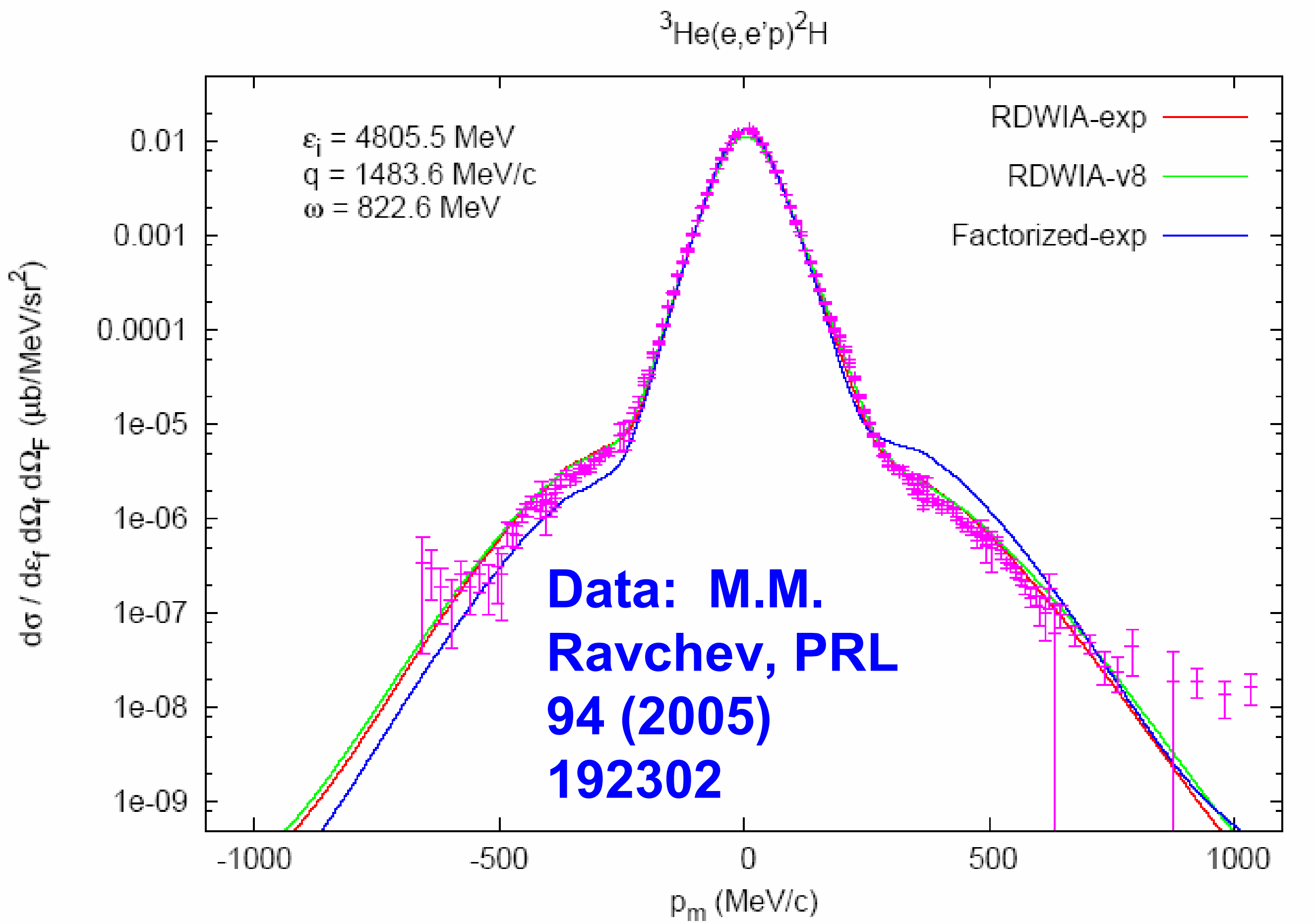


Factorization approach

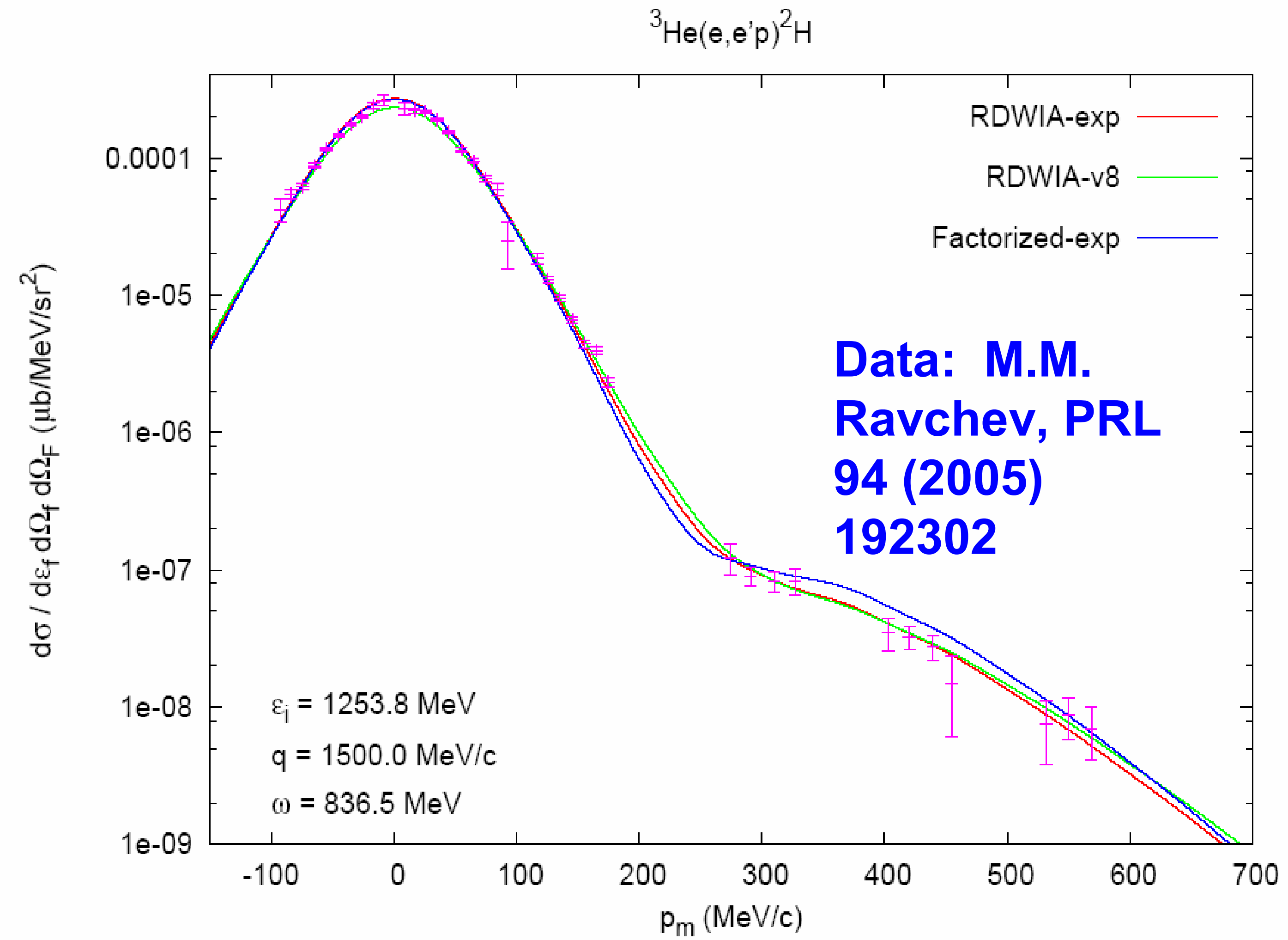
- Under factorization, the cross-section splits completely into an elementary boson-nucleon part, depending on the interaction, and a nuclear part (spectral function) that depends on E_m and p_m and that is completely independent on the nature of the probe
- The single-nucleon responses are in principle different for each kind of boson, due to different structure of the elementary current operator, but the nuclear response is the same if factorization is (nearly) recovered
- Let's put it in another way: The properties of the nuclear response are (to a large extent) independent on the probe that excites such response, if factorization is recovered
- The applicability of nuclear responses from one reaction to another depends on the extent that factorization is fulfilled



RMF: Small non-factorization (non-EMA, LS) effects in the cross-sections for moderate p_m

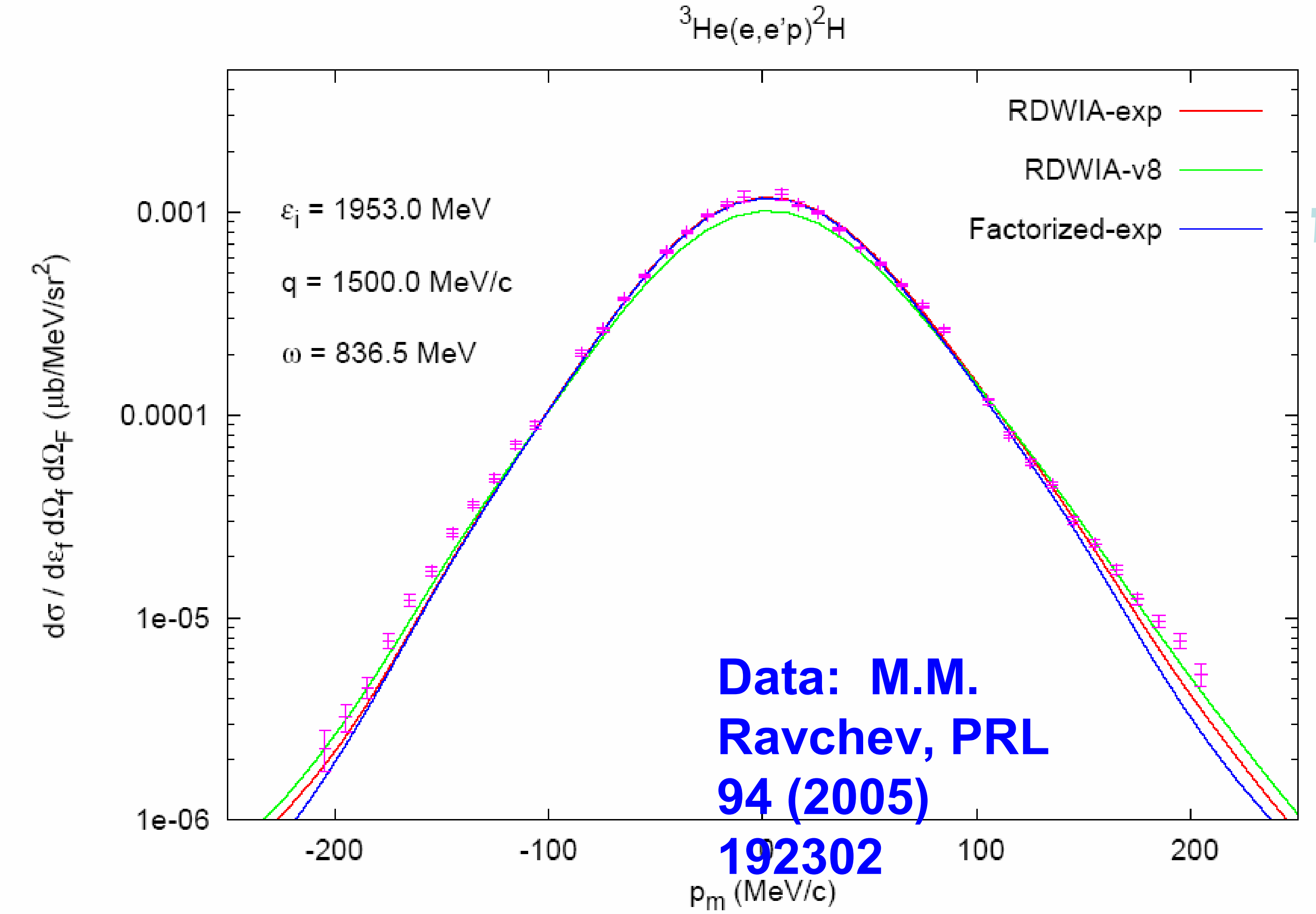


Breakdown of factorization will be moderately seen at moderately demanding kinematics (moderately high p_m)



Breakdown of factorization will be moderately seen at moderately <u>demanding</u> kinematics (moderately high p_m)

NUCLEAR RESPONSE TO MULTI-GEV NEUTRINOS IN THE RELATIVISTIC MEAN FIELD MODEL

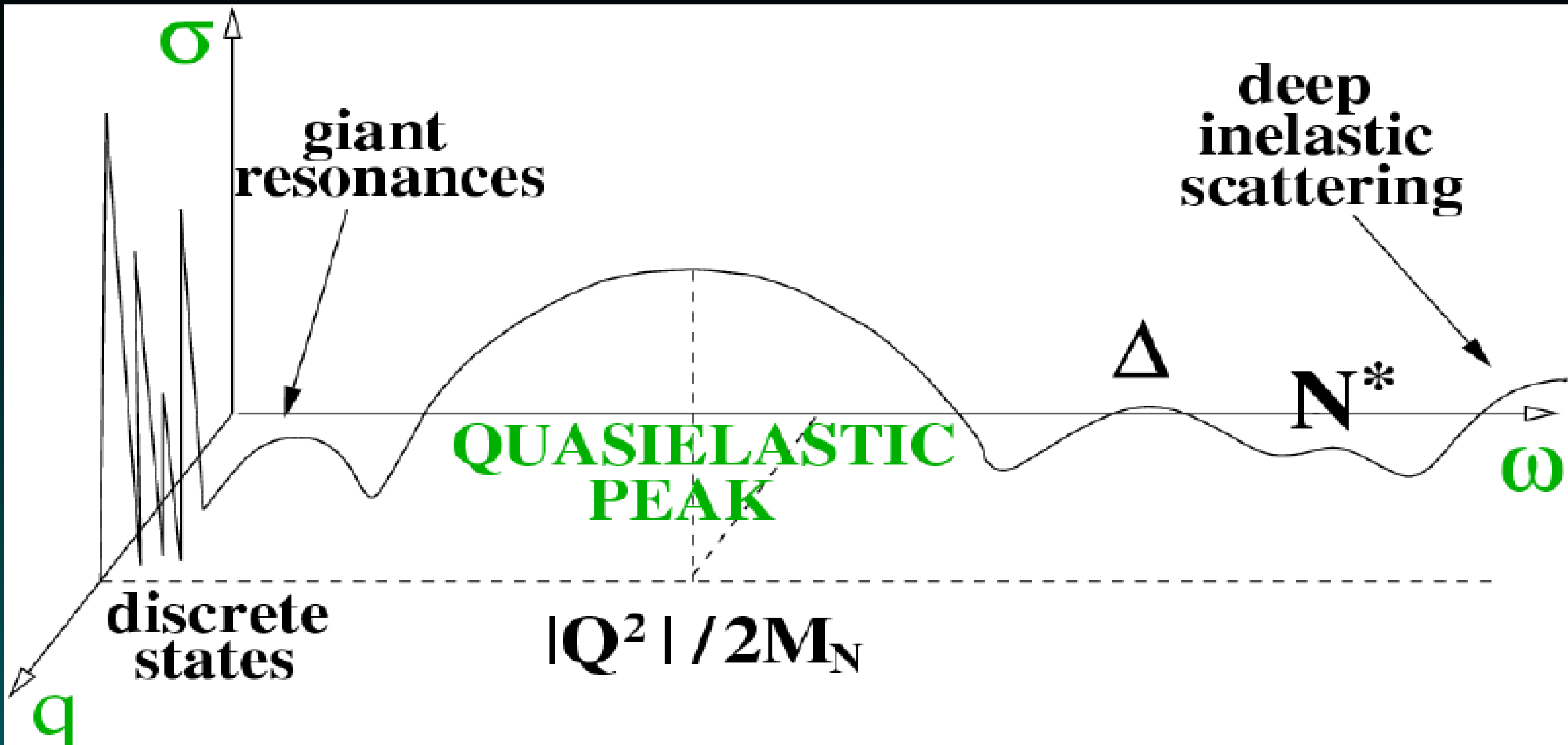


Breakdown of factorization will be moderately seen at moderately <u>demanding</u> kinematics (moderately high p_m)

For inclusive scattering at QE kinematics and multi-GeV energies

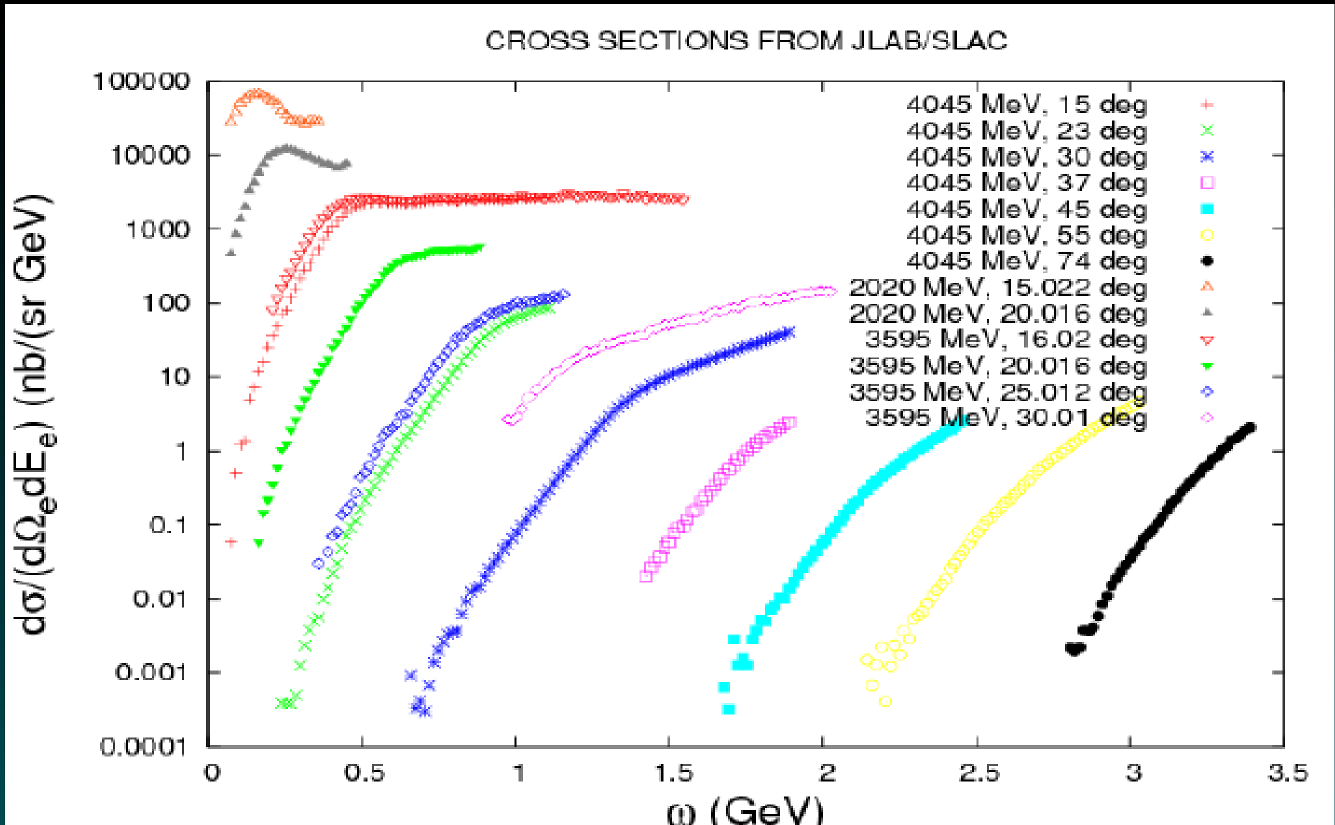
- Small breakdown of factorization in the cross-section
- The inclusive cross-section can be understood from the product of the nuclear response (spectral function) and the elementary probe-nucleon response
- To the extent that factorization is (aproximately) fulfilled, the elementary probe-nucleon response and kinematical factors are factored out and the remaining nuclear response is the same for same nucleus, q and ω transfer, irrespectively of the nature of the probe

Inclusive electron scattering on nuclei



Many things may happen to the nucleus, depending on the values of q and ω

Inclusive ^{12}C quasielastic electron data



Day *et al*, PRC48(1993)1849

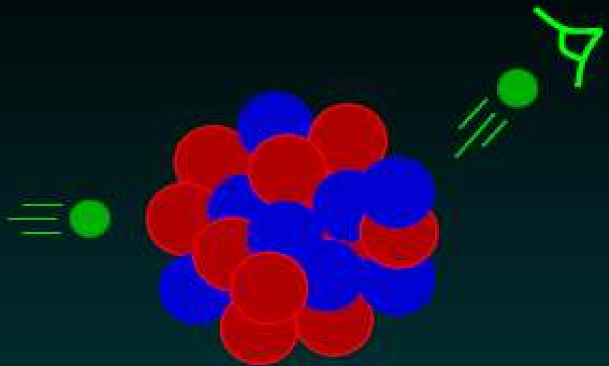
Arrington *et al*, PRL82(1999)2056

$0.5 < q < 4$ GeV/c

Can we organize *inclusive* nuclear cross-section data?

$$f(q,\omega) \propto \frac{\left[\frac{d\sigma}{d\Omega_e dE_e}\right]}{\overline{\sigma}_{electron-nucleon}}$$

Some general ideas about scaling in inclusive scattering



- Requires a **weakly interacting probe** and a **composite target**
- The probe must scatter from **one of the bound constituents** of the target

$$F(q,\omega) = \frac{\left[\dfrac{d\sigma}{d\Omega_{probe} dE_{probe}}\right]}{\overline{\sigma}_{probe-constituent}}$$

At high q this function depends on a combination of q and $\omega \Rightarrow$ **SCALING**

y-scaling variable?

y is the minimum initial momentum of the nucleon allowed by the kinematics.

$$y \approx \sqrt{\omega(2M_N + \omega)} - q$$

$$\text{If } y = 0 \Rightarrow$$

$$q^2 = \omega(2M_N + \omega) \Rightarrow$$

$$|Q^2| = 2M\omega$$

y and Bjorken x scaling variables are closely related to each other! One has binding energy and nucleus recoil corrections

We will use $\psi = y/k_F$ as scaling variable

Scaling: An important simplification (I)

- If we were dealing with scattering off free nucleons, the nuclear response would not depend on q and ω in an independent fashion, rather it would depend only on Q^2 (or any function of Q^2)

- As the nucleons in the nucleus are bound and strongly interacting (i.e., they are off mass-shell), a certain dependence on both q and ω and not only Q^2,may be expected. This extended dependence can be also due to FSI

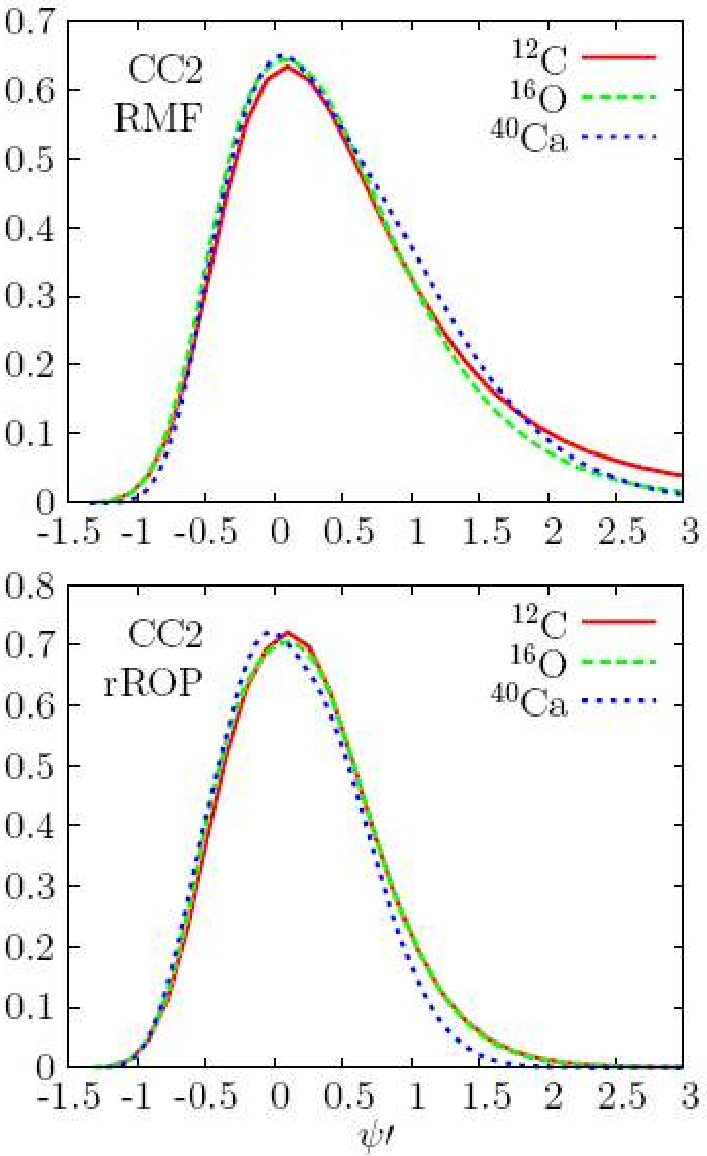
Scaling: an important simplification (II)

- Most models that deal with nucleons are barely off-shell (if at all) and either they have no FSI, like the Fermi Gas (relativistic or not) or they have relatively weak FSI, as most nonrelativistic models. They do not display significant dependences on the scaled response other than Q^2. It is not surprising that these models exhibit *scaling of first kind*

- The variability in the nuclear species, once factored out the single-proton and single-neutron response, can be described by only one parameter, the Fermi momentum. As this density dependence can be factored out very effectively in most models, all nuclei display the same k_F-scaled 'nuclear' response. This is *scaling of second kind*

For inclusive scattering at QE kinematics and multi-GeV energies

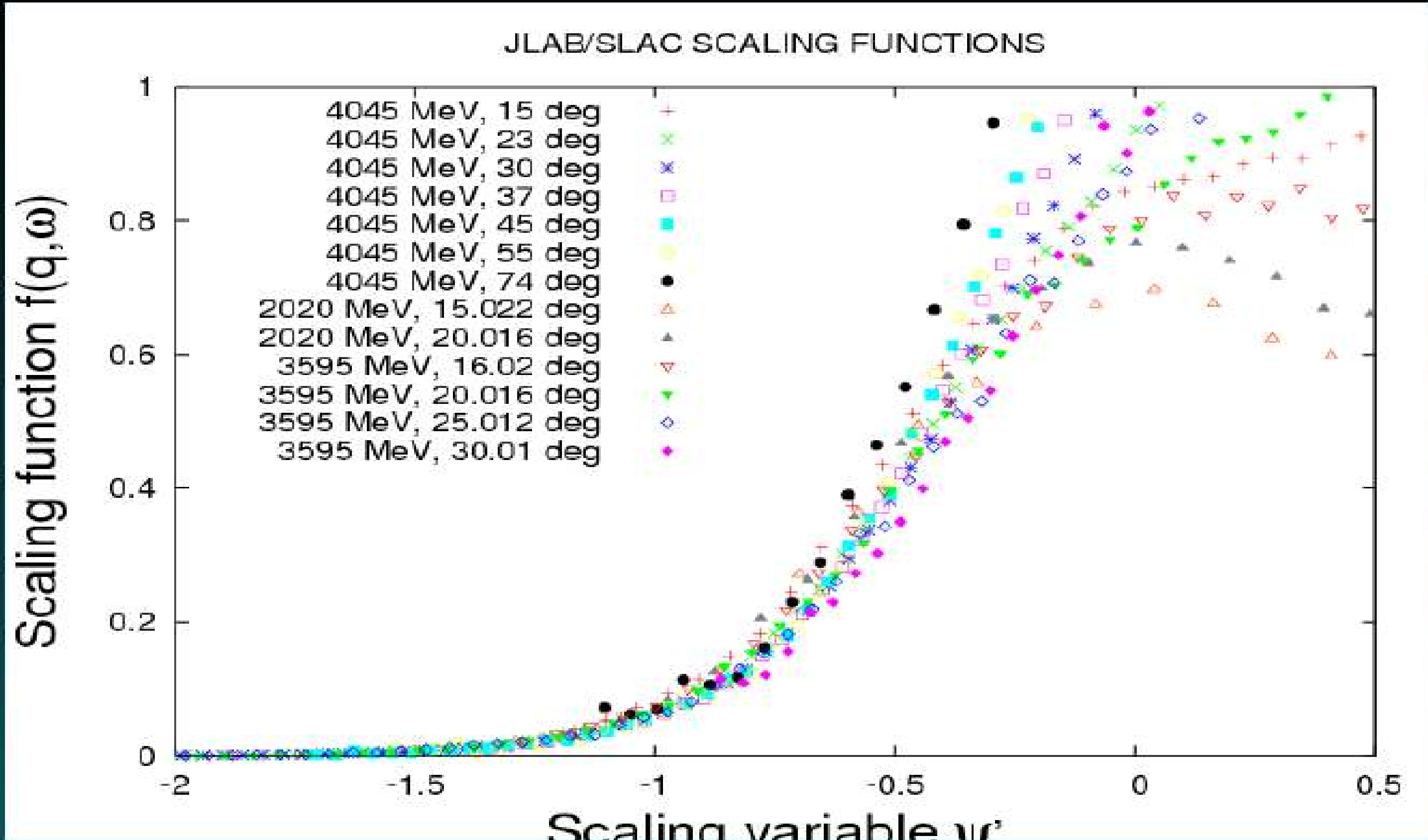
•Factorization is a pre-requisite for scaling that is not strictly fulfilled if strong potentials are present, due to the enhancement of the lower components, dispersive effects and LS terms but, in fact, the factorization approach works quite well for cross-sections under QE conditions, OBE and IA, even for a model like RMF



Both scaling of first and second kind are clearly observed in the predictions of theoretical models based upon OBE+IA. Even when these models are (un)factorized or when they include important FSI interactions among nucleons. This has to do with the properties of the (distorted) nuclear response in general and not with the properties of the probe (provided OBE). Thus, the scaling features observed in electron scattering are also expected in neutrino (charged and current) scattering

Inclusive ^{12}C quasielastic electron data

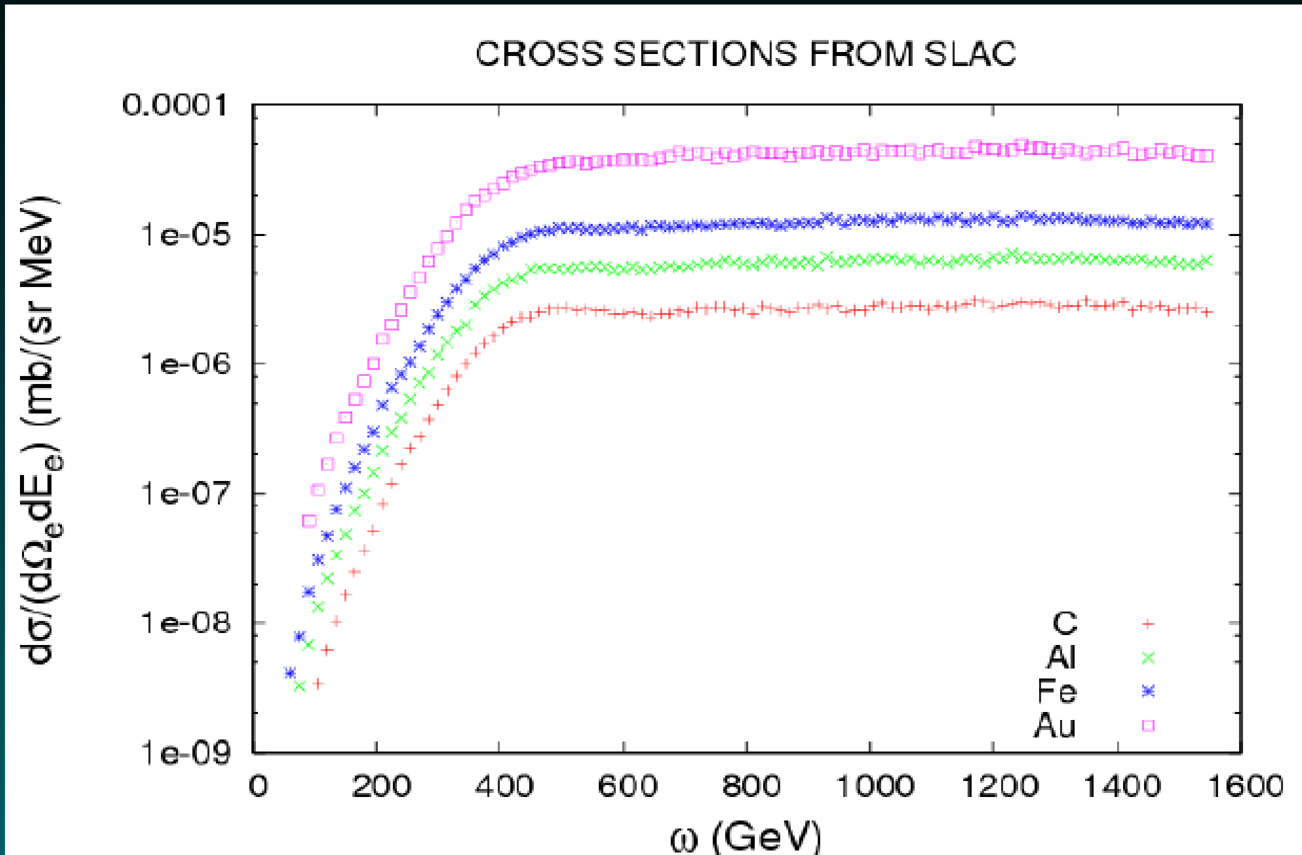
SCALING BEHAVIOR



Quite good scaling for negative scaling variable (y-scaling). Large violations for large energy transfers due to the transverse response

More inclusive quasielastic electron data

Same transferred momenta, different targets (C, Al, Fe, Au)

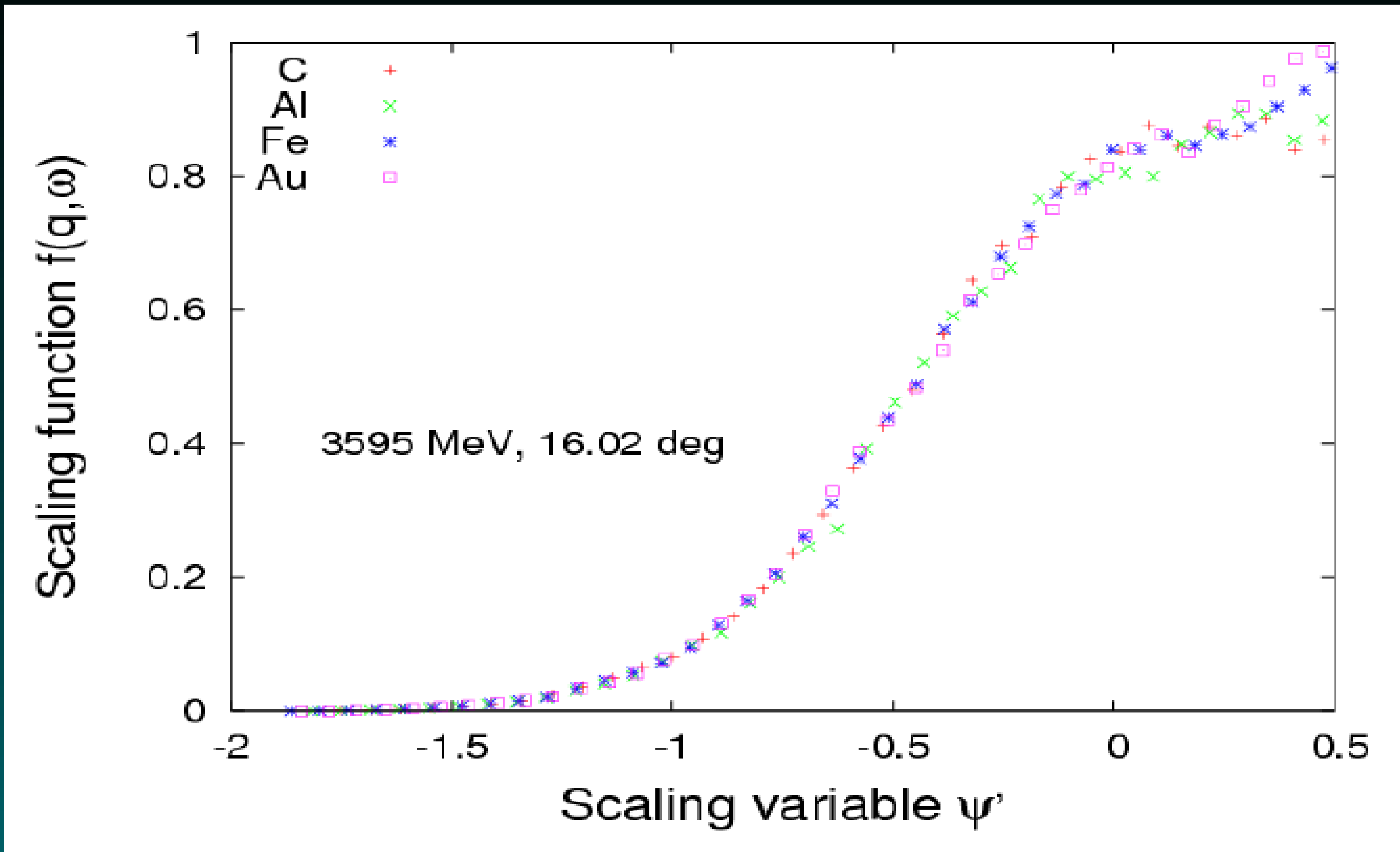


Day *et al*, PRC48(1993)1849

$q \approx 1$ GeV/c
$E_e = 3{,}6$ GeV,
$\theta_e = 16^o$

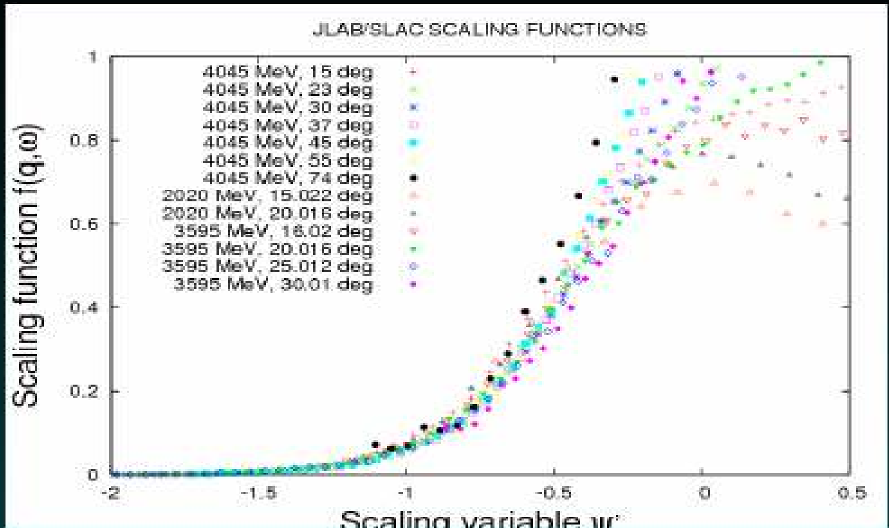
Inclusive quasielastic electron data at $q \approx 1$ GeV/c

SCALING BEHAVIOR



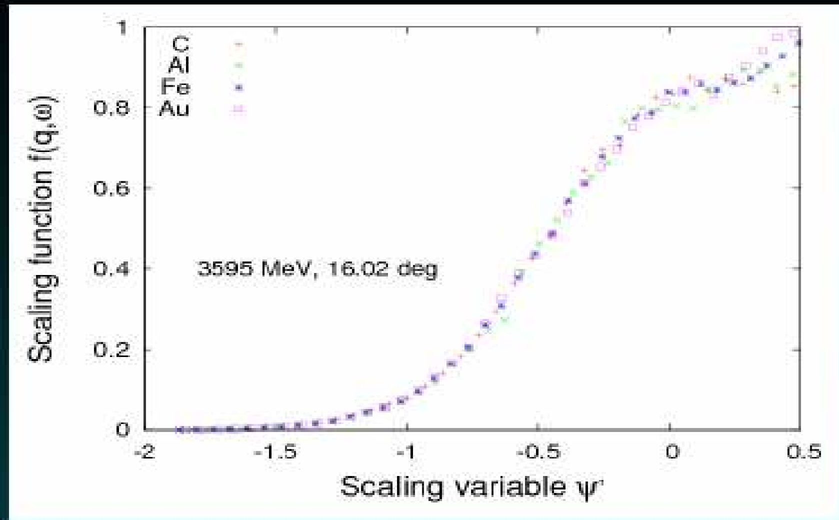
As a summary

Same target (^{12}C), different transferred momenta



FIRST KIND SCALING

Same transferred momenta, different targets (C, Al, Fe, Au)



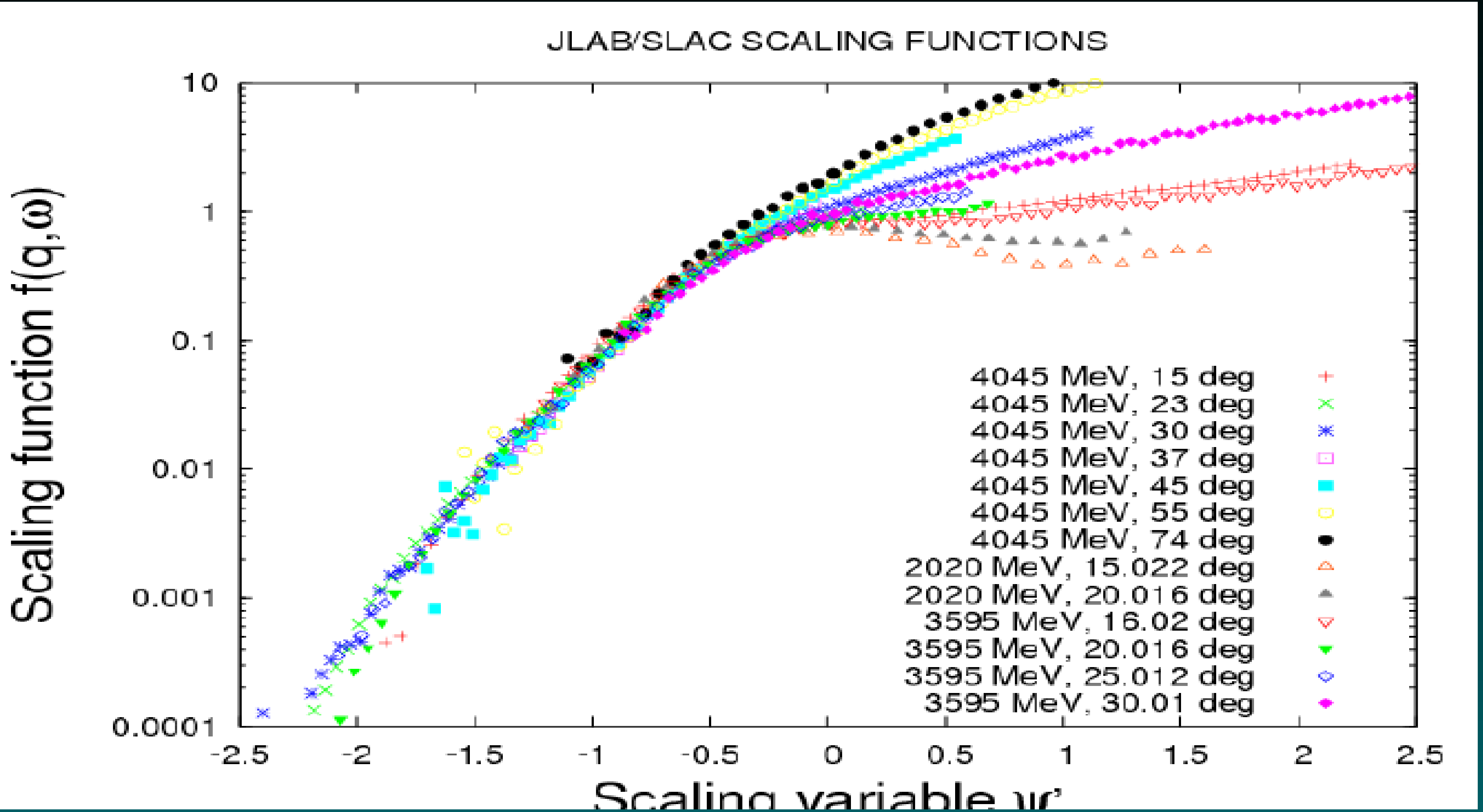
SECOND KIND SCALING

FIRST (y-scaling) + SECOND = SUPERSCALING

Day *et al*, Ann. Rev. Nucl. Part. Sci. **40** (1990) 357,
Donnelly and Sick, Phys. Rev. C **60** (1999) 065502,
Donnelly and Sick, Phys. Rev. Lett. **82** (1999) 3212

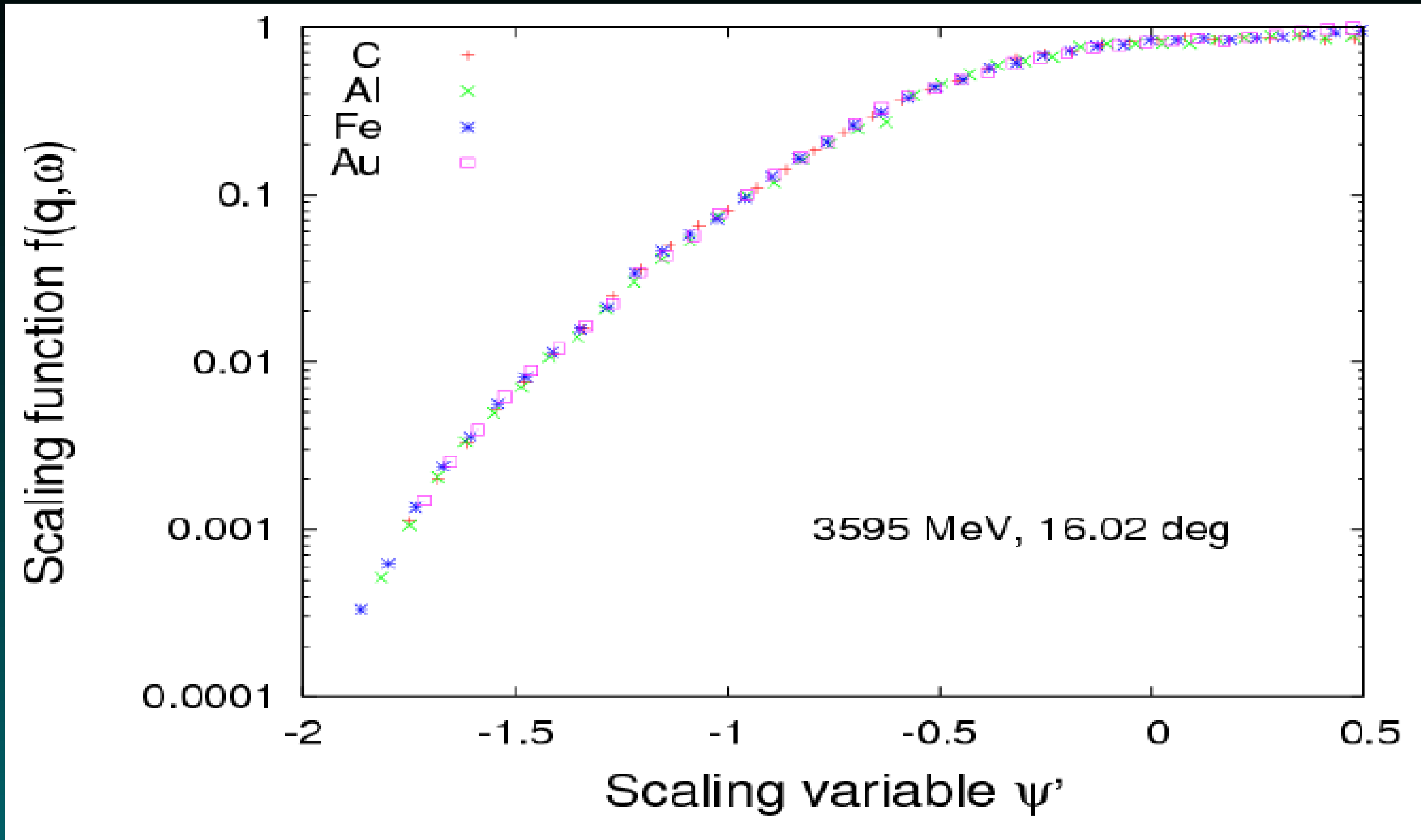
Inclusive ^{12}C quasielastic electron data

SCALING BEHAVIOR IN LOG SCALE



Inclusive quasielastic electron data at $q \approx 1$ GeV/c

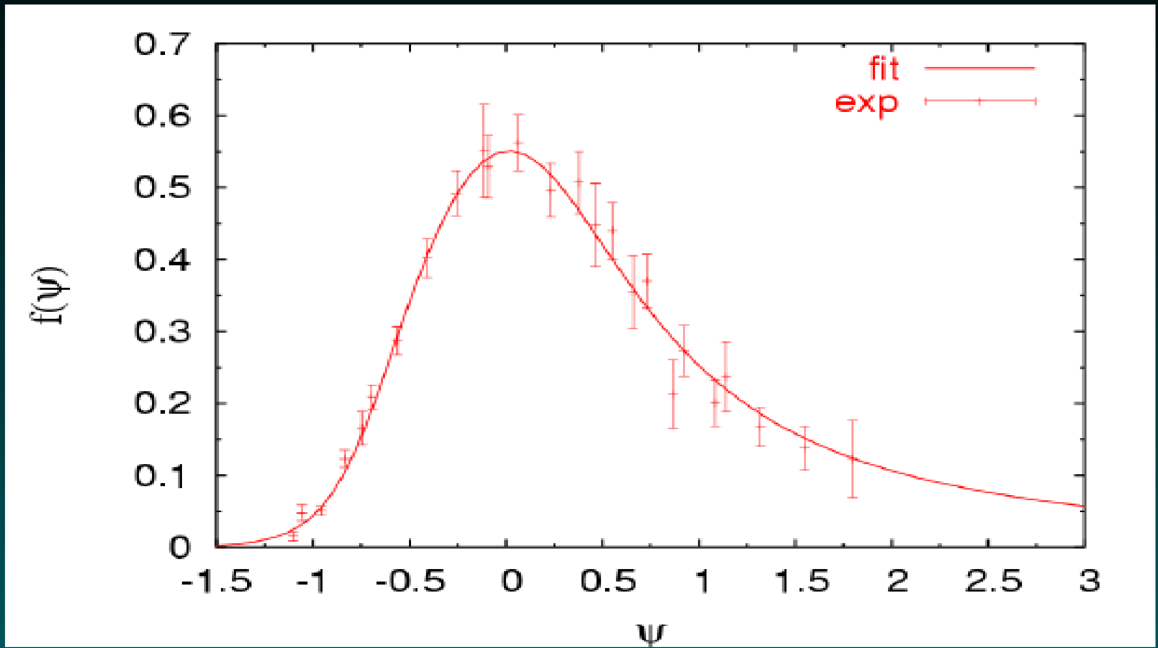
SCALING BEHAVIOR IN LOG SCALE



A phenomenological (super-)scaling function arises

Inclusive electron data exhibit superscaling behaviour in the QE (and also in the Δ peak) region

$\Rightarrow$ **we have a phenomenological scaling function**



(e, e') **DATA** $\Longleftrightarrow$ **SCALING FUNCTION**

NUCLEAR RESPONSE TO MULTI-GEV NEUTRINOS IN THE RELATIVISTIC MEAN FIELD MODEL

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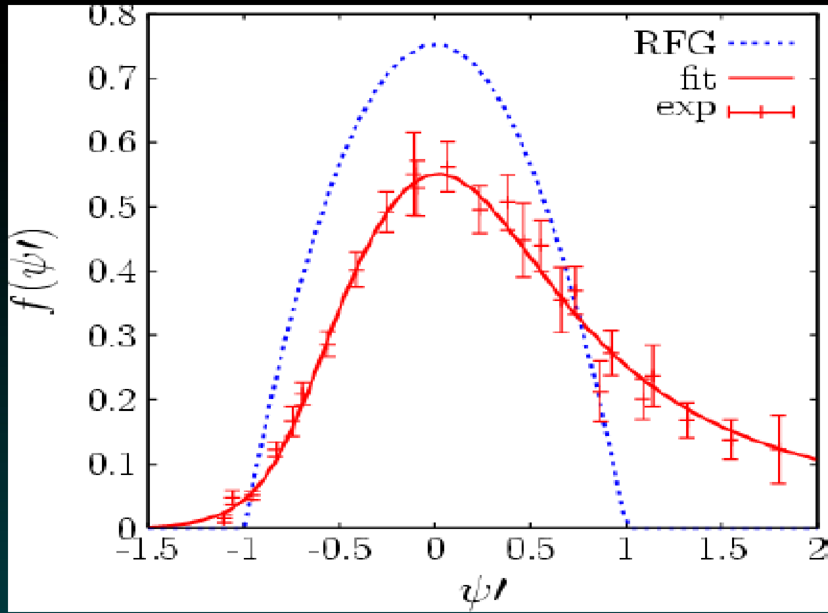
Relativistic Fermi Gas:

Perfect superscaling behaviour

(Alberico *et al*, Phys. Rev. C 38, 1801 (1988))

Experimental data:

Good superscaling behaviour, although not perfect



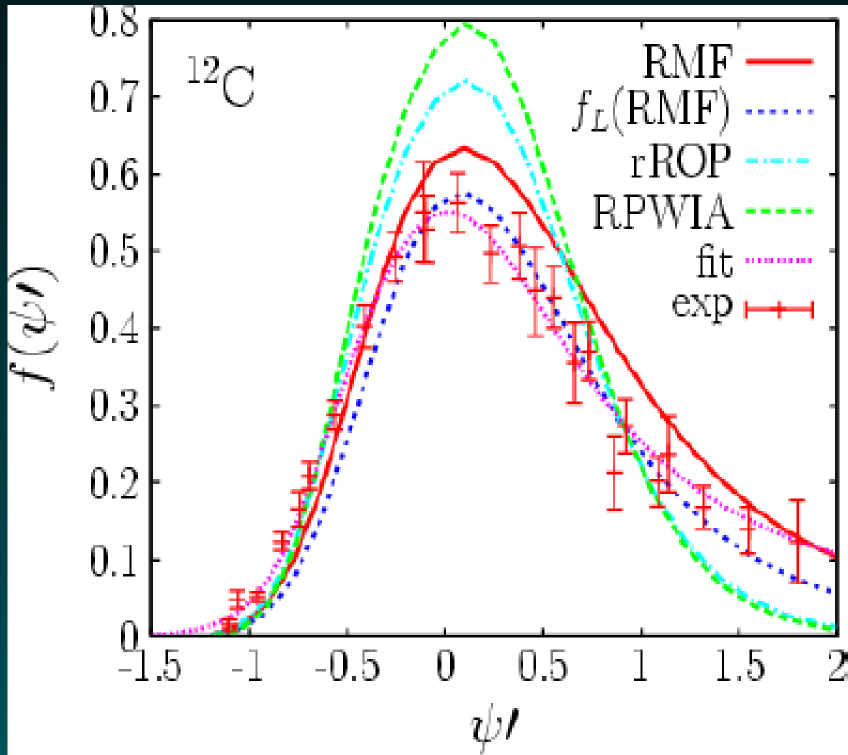
The RFG superscales, and data also superscale, but the scaling functions of data and of RFG disagree $\Rightarrow$

The RFG lacks important initial and final state nuclear dynamics effects, even at high energies!

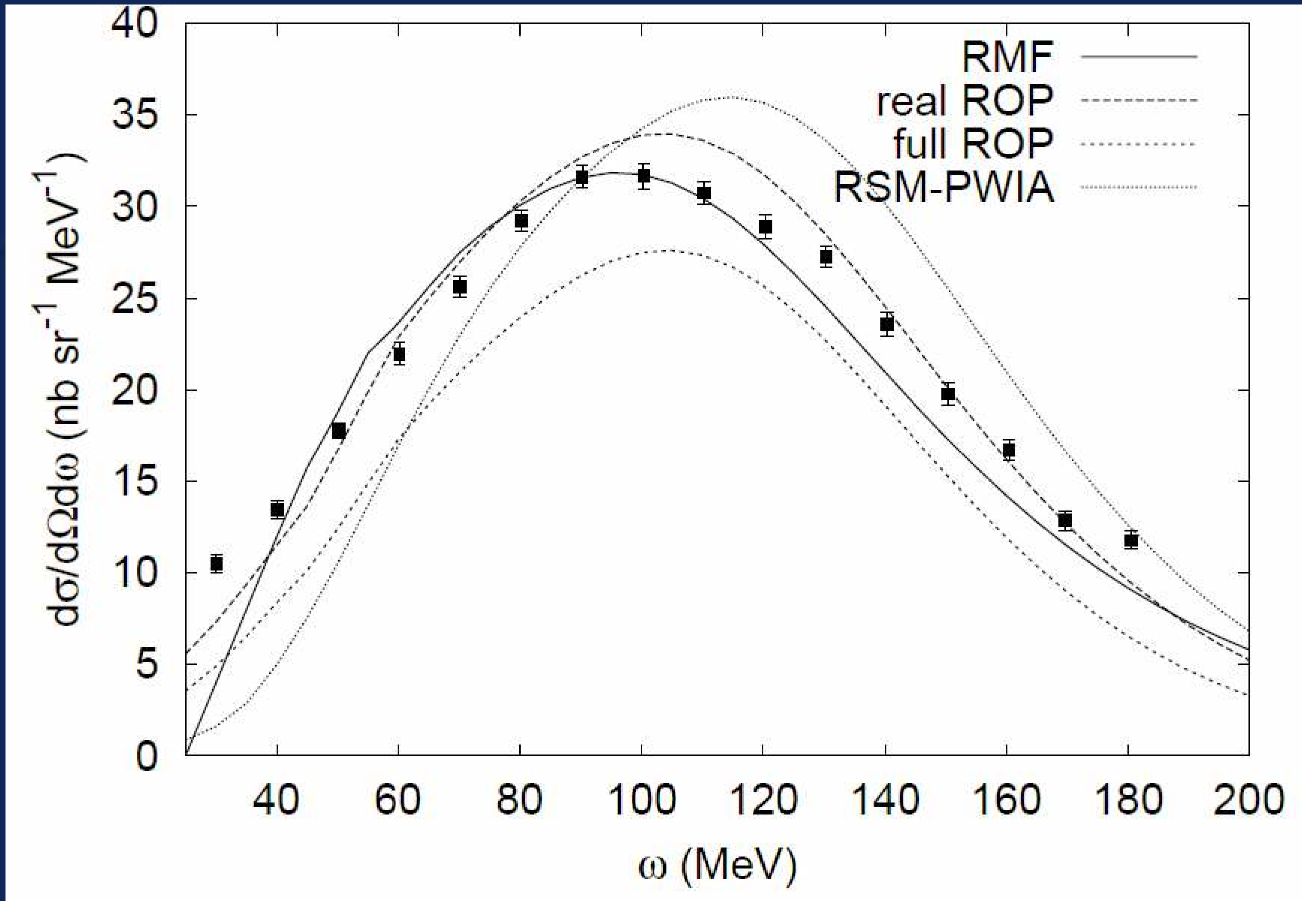
...not many models have been able to reproduce the experimental scaling function...

The **RELATIVISTIC IMPULSE APPROXIMATION + RELATIVISTIC MEAN FIELD (RIA-RMF)** for describing the bound and ejected nucleon does, both in magnitude and shape

(Caballero et al,
Phys. Rev. Lett. **95**, 252502 (2005),
Phys. Rev. C **74**, 015502 (2006)...)

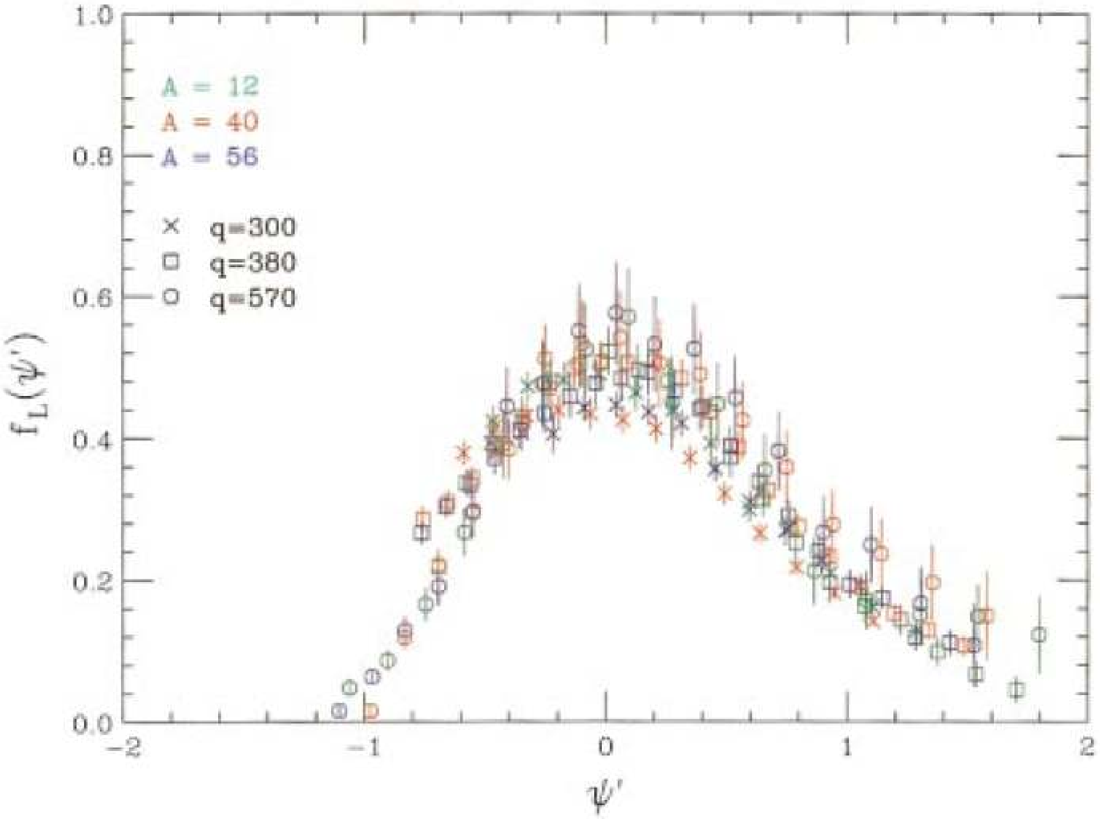


RMF results compare well with (e,e’) data also at moderate momentum transfer: ^{12}C(e,e’) $|\mathbf{q}| \simeq$ 400 MeV/c



NUCLEAR RESPONSE TO MULTI-GEV NEUTRINOS IN THE RELATIVISTIC MEAN FIELD MODEL

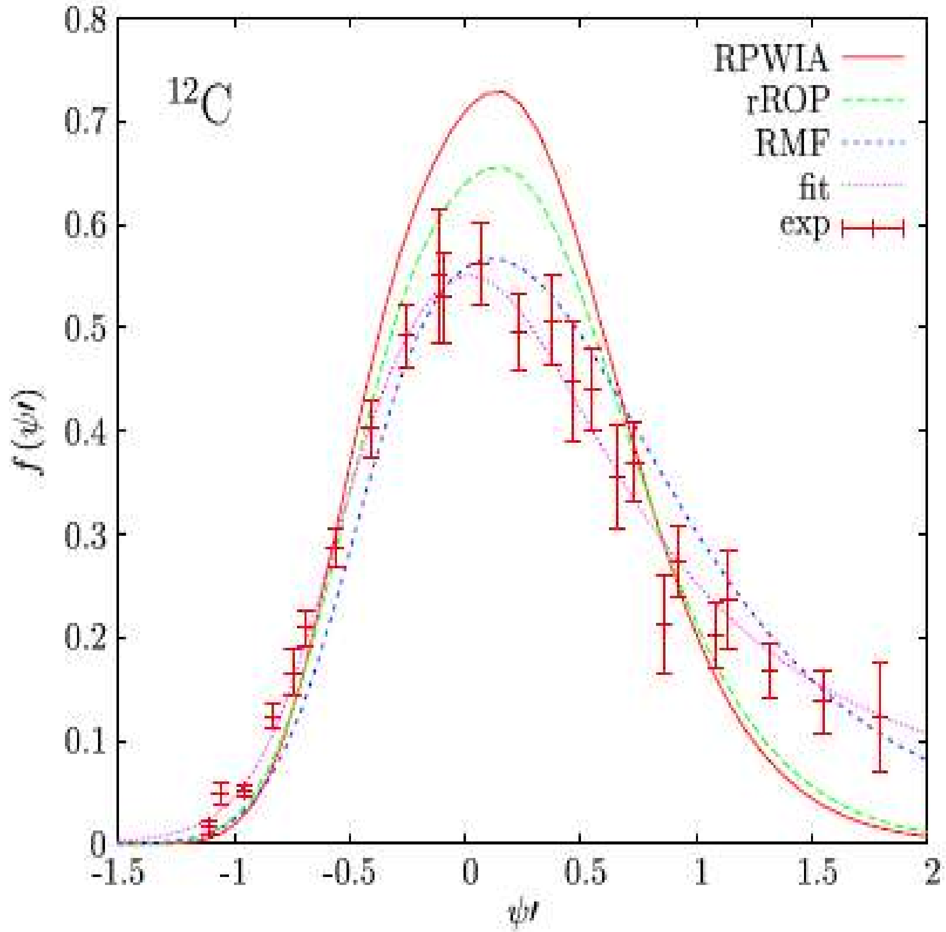
Both scaling of first (mild) and second (good) kind are clearly observed in the inclusive electron scattering data. This supports the theoretical predictions indicating that off-shellness of the nucleons in nuclei and even strong FSI do not destroy scaling



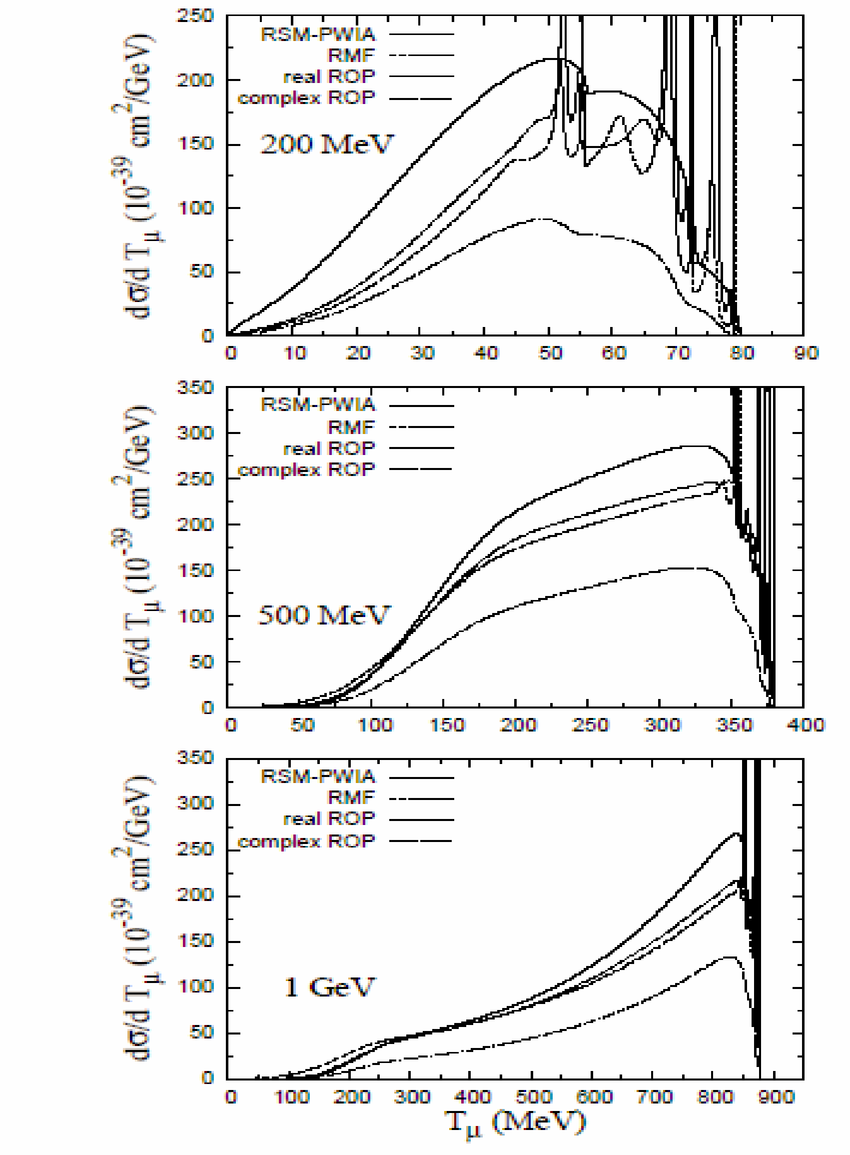
This allows to extract a (*exceedingly convenient!)* universal superscaling function from the inclusive electron scattering data

It is clear that the longitudinal data exhibit a large asymmetric tail

NUCLEAR RESPONSE TO MULTI-GEV NEUTRINOS IN THE RELATIVISTIC MEAN FIELD MODEL



- |q|=1 GeV/c
- We isolate the pure nucleonic response by comparing to the L-scaling function
- More symmetrical responses are ruled out. RMF compares better with data
- Use RMF to predict neutrino-nucleus cross-sections

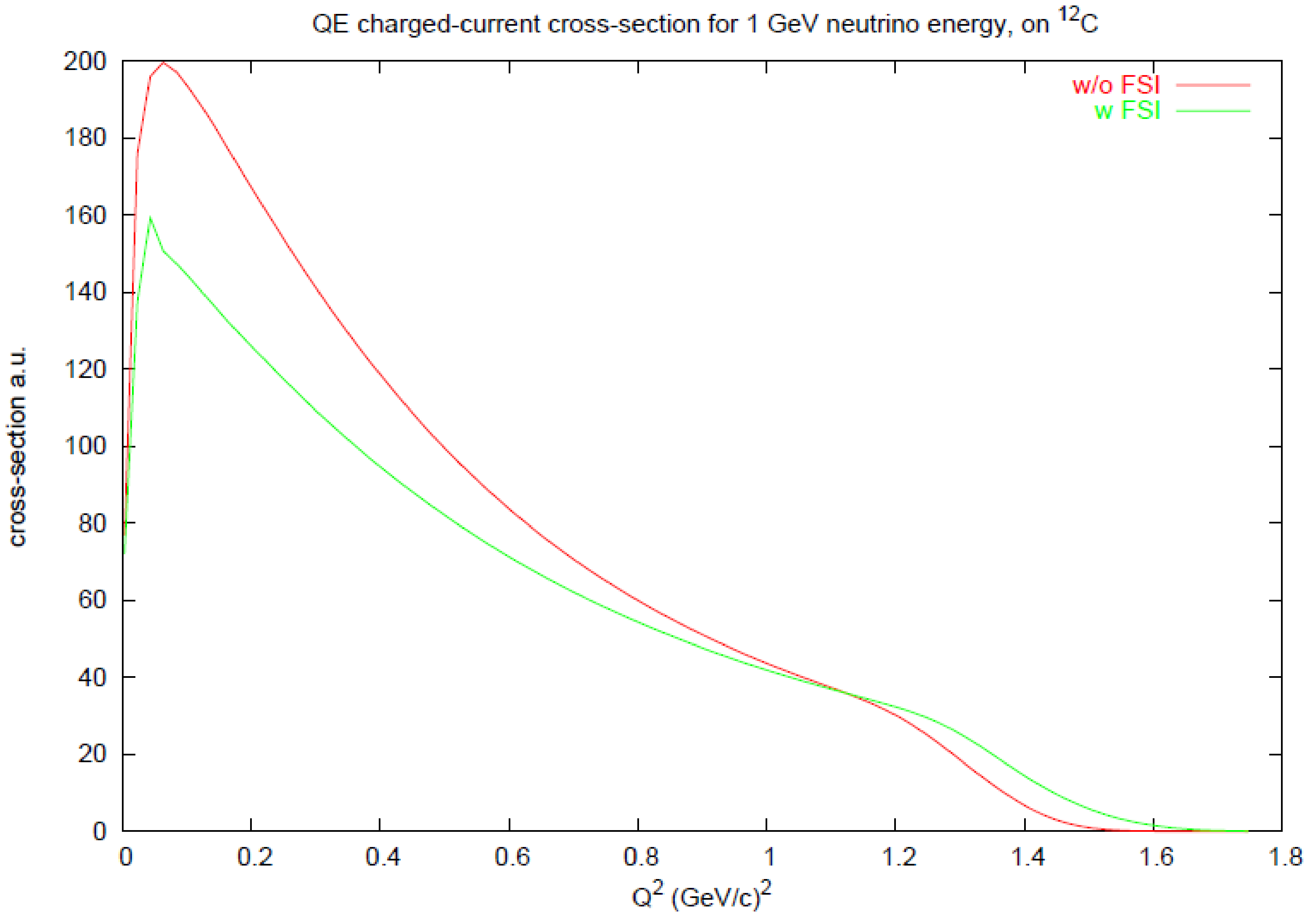


RMF Predictions for neutrino reactions

Charged currents: There are sizeable effects of FSI even at 1 GeV

$^{12}C(\nu,\mu^-)X$

C. Maieron et al., PRC68 (2003)048501



RMF calculations show that FSI have an effect even for 1 GeV neutrino energy

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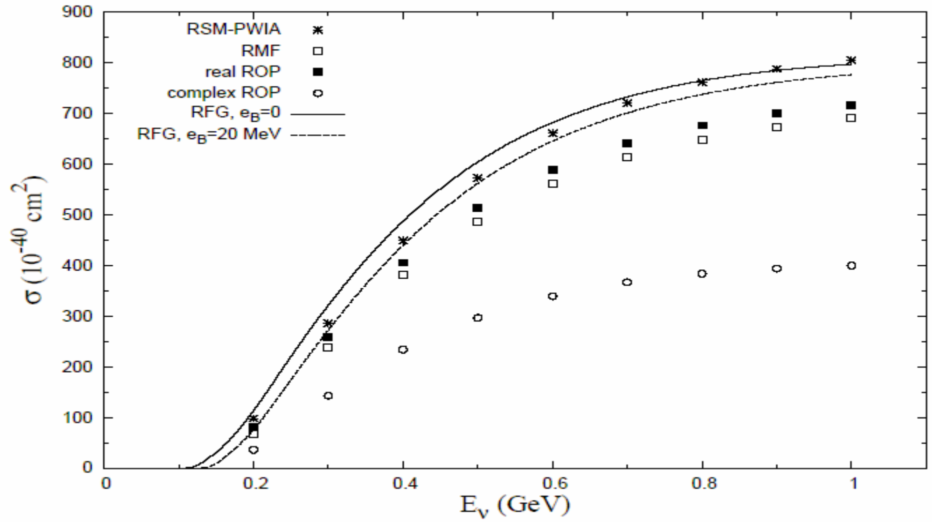


FIG. 4: Integrated cross section $\sigma(E_\nu)$ for the quasielastic scattering of muon neutrinos on ^{16}O as a function of the incident neutrino energy. The curves are calculated within the RFG model with $k_F = 225$ MeV and binding energy $e_B = 0$ (solid line) and $e_B = 20$ MeV (dashed). The points correspond to RSM calculations without FSI (stars) and with FSI effects taken into account within the RMF (empty squares), real ROP (full squares) and complex ROP (circles) approaches.

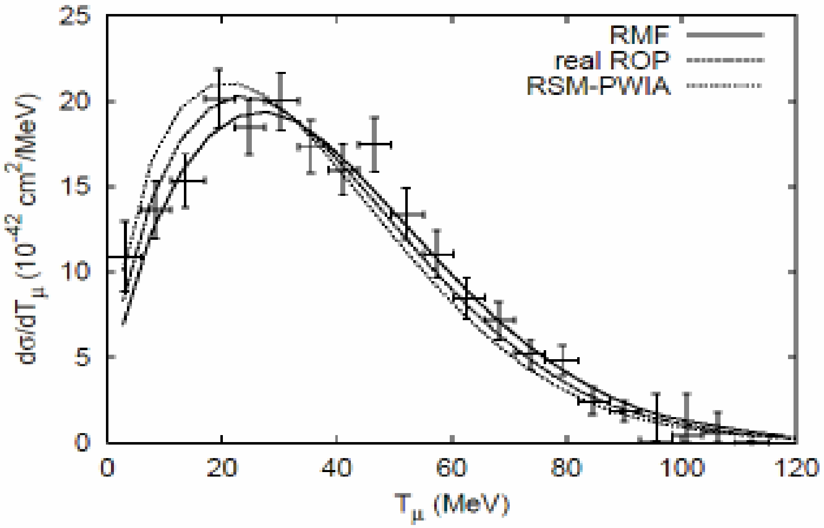


FIG. 5: Observed distribution of muon kinetic energies T_μ compared with the flux–averaged predictions of our RSM, in PWIA (dotted line) and including FSI within the RMF (solid) and purely real ROP (dashed) frameworks. The theoretical distributions have been normalized to give the same integrated values as the experimental points, and have been folded in energy with a bin size of 5 MeV, the same employed for the experimental data. Data are from Albert et al. [12].

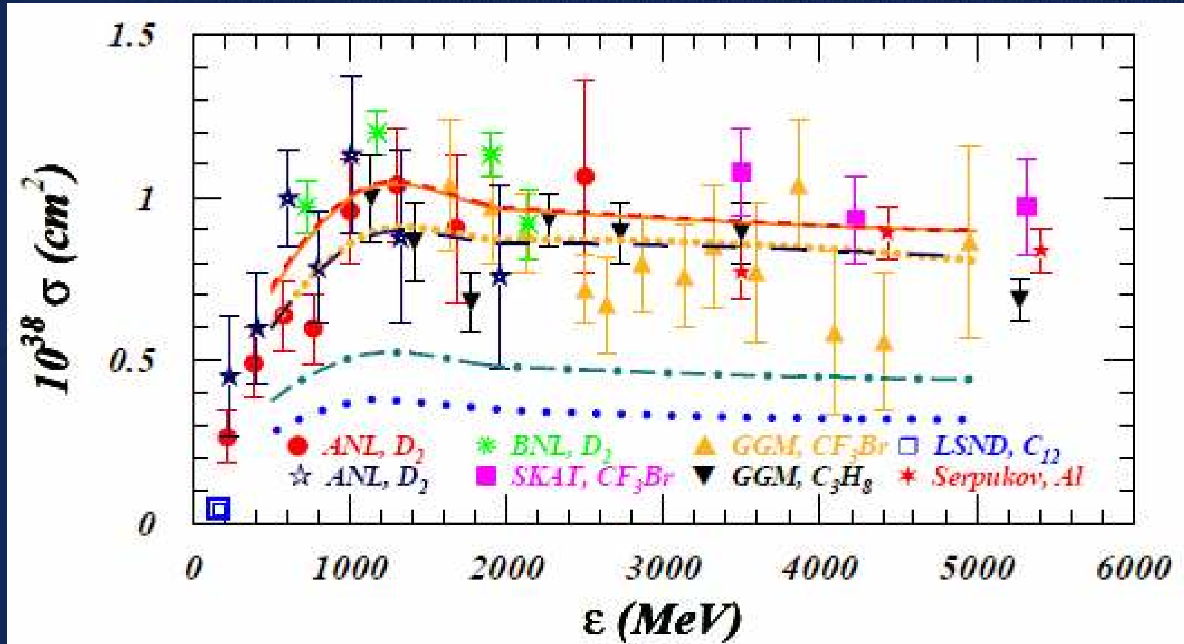
Comparison to LSND $^{12}C(\nu,\mu^-)X$ data

σ_{RMF}=(15 to 16) x 10^{-40} cm^2 that is 40% above data. MEC shall reduce this by no more than 10%

Y. Umino et al. PRC 52 (1995) 3399

NUCLEAR RESPONSE TO MULTI-GEV NEUTRINOS IN THE RELATIVISTIC MEAN FIELD MODEL

RMF predictions for nucleonic contributions to CC and NC neutrino scattering can be done



Total CC predictions for non-pionic 'quasielastic' charged current reactions (ν,μ^-) obtained: a) without FSI interactions (red curve). With FSI interactions within RMF for ^{12}C and ^{56}Fe (dotted orange and long dashed blue lines, respectively). 'Pure' elastic contribution is shown by dot-dashed (green, ^{12}C) and long dotted (cyan, ^{56}Fe) curves. Data from several experiments and targets are also plotted. 10% effect of FSI can be observed, even at 5 GeV

M.C. Martínez et al., PRC73 (2006) 024607

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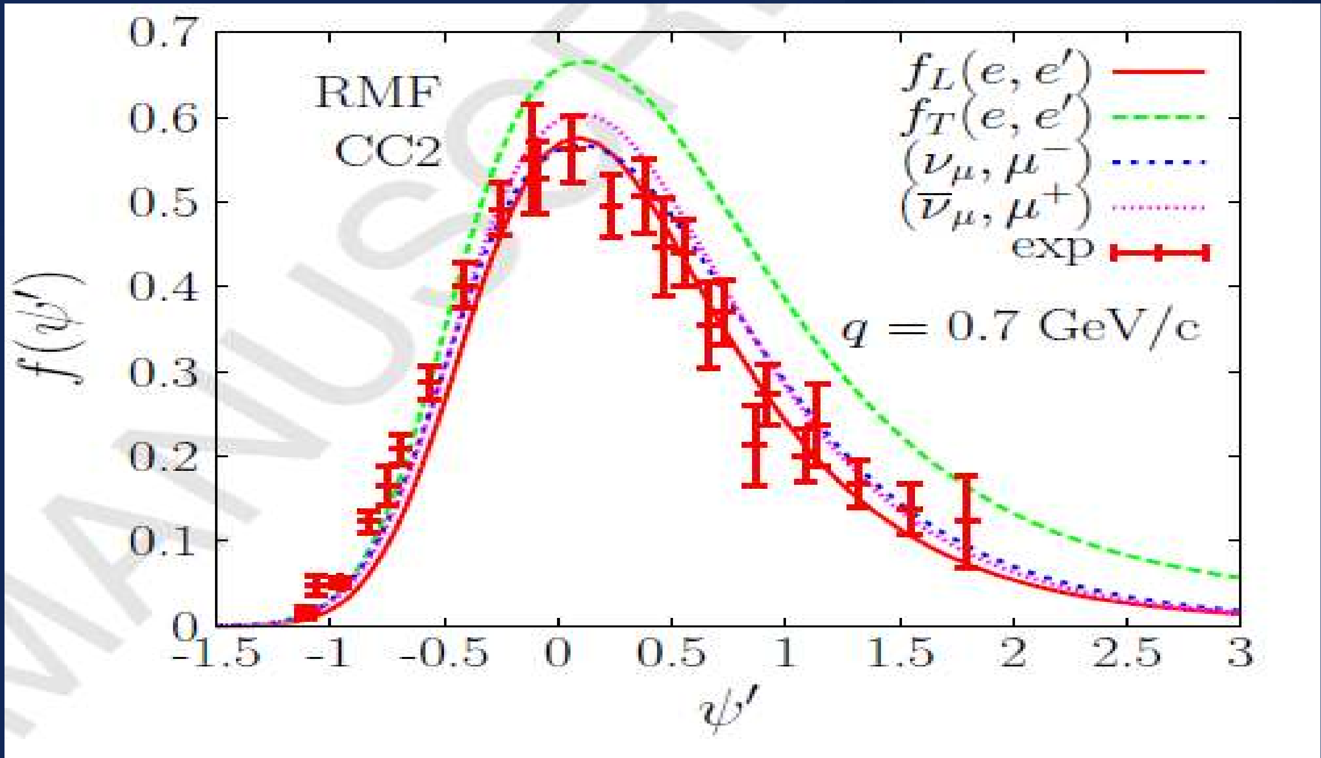
The theoretical analysis indicates that the universal superscaling function is, to a very large extent, independent on the probe. This comes from actual calculations for electron, charged and neutral currents within RMF model. Scaling of *THIRD* kind?



Use scaling to predict neutrino-nucleus cross-section. It’s easy for charged currents

For neutral ones?

Procedure to check validity (or not) from scaling:

1. Get our inclusive NC neutrino-nucleus cross sections ($d\sigma/dE_N d\Omega_N$) by integrating over the scattered neutrino variables our exclusive cross sections.

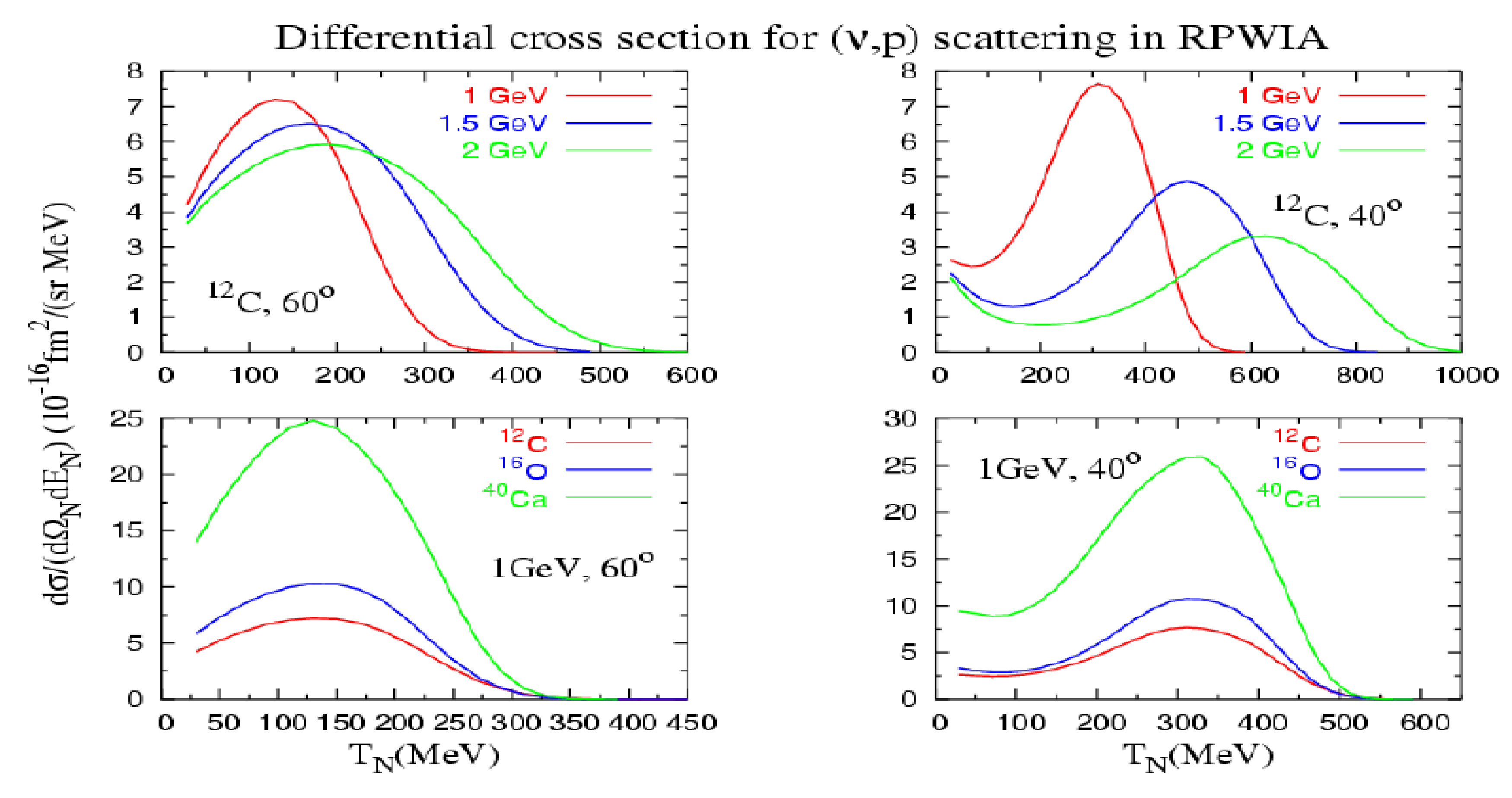
2. Divide by an averaged (over ϕ') NC neutrino-nucleon cross section. Multiply by the Fermi momentum k_F of the nucleus to make the result adimensional in order to check scaling of first and second kind.

$$f(q', \Psi') = k_F \frac{\left[\frac{d\sigma}{d\Omega_N dE_N}\right]}{\overline{\sigma}_{sn}^{NC}}$$

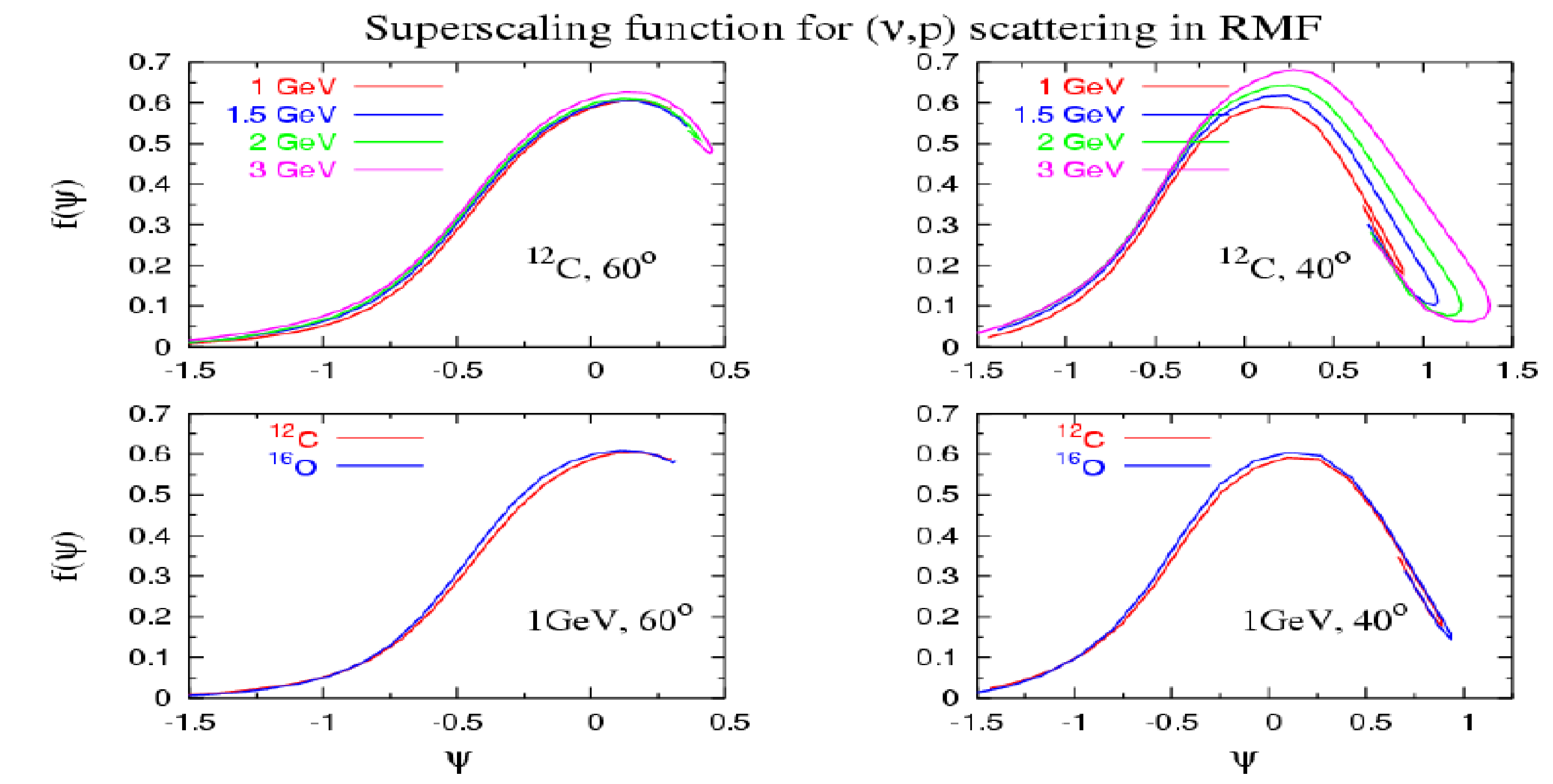
3. Represent the result as a function of a NC scaling variable Ψ', derived in Phys. Rev. **C** 73, 035503 (2006), inspired in the RFG.

Note: All the responses (L, T, TL, TT, T', TT') will contribute to our u-inclusive cross section!

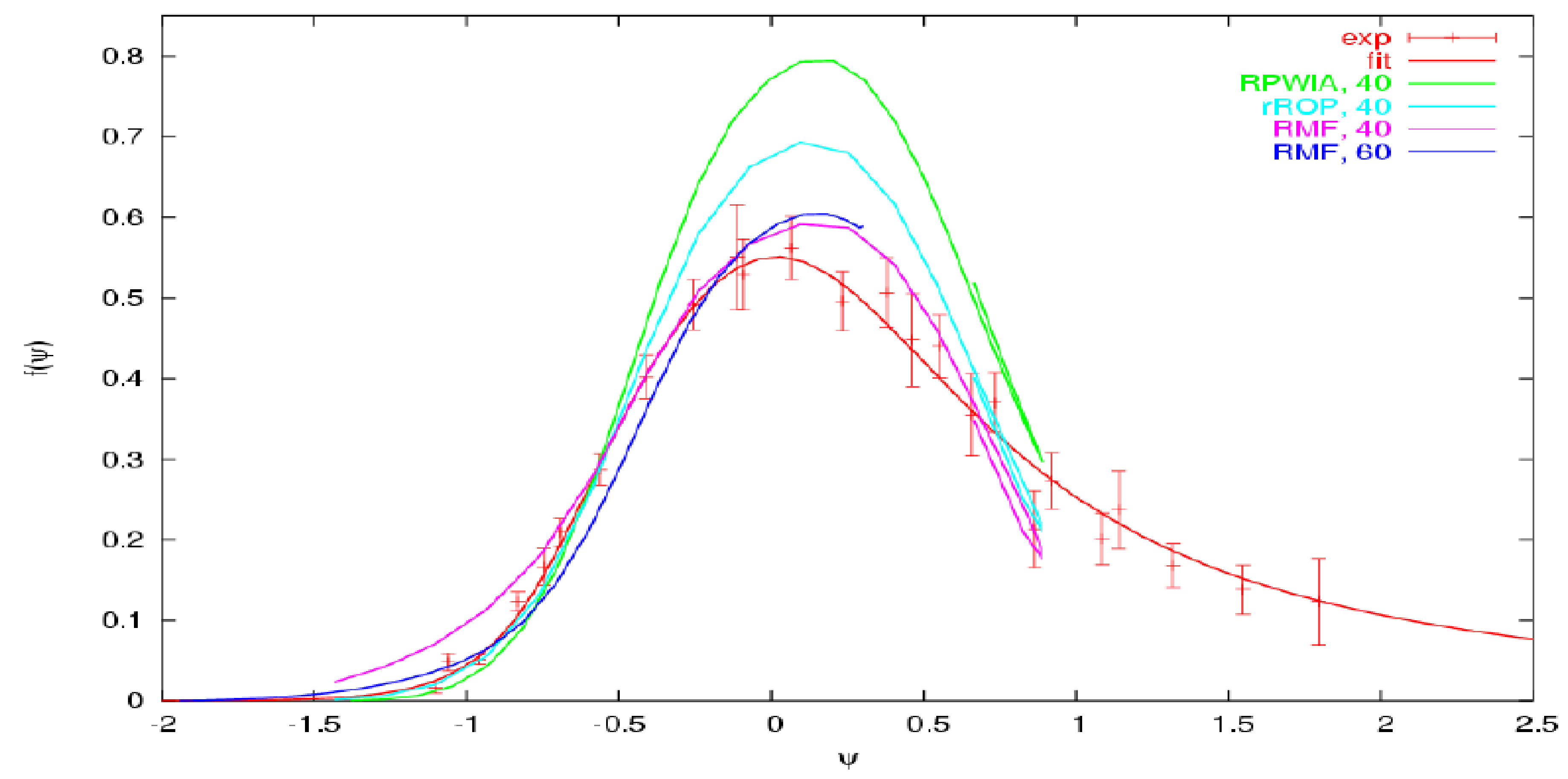
Results: Cross sections for different beam energies and nuclei for neutrinos



Results: Superscaling within RMF



Comparison to (e, e') averaged experimental function



CONCLUSIONS

•RMF agrees very well with the L-scaling function, what indicates that it is suitable to predict the nucleonic contribution to the nuclear response to electron scattering in the QE regime

•RMF exhibits scaling (and slight departure from it) of first and second kind (and also zero kind) at a level similar to the one allowed by the data. Further extensive comparisons of RMF to available electron scattering data show good agreement for the pure nucleonic responde in the 0.3-2.5 GeV/c q range. RMF can be used to test the validity of other approaches, such as factorization and scaling

•RMF, ins spite of off-shell nucleons and strong FSI, shows good scaling of third kind, that is, universality of nucleonic response to weak interaction probes

•For the neutral current case with u-channel kinematics, scaling is a good approximation for not too small (>60º) nucleon scattering angles

•If or when scaling (response depends on one variable) fails, factorization can be employed to predict neutrino cross-sections from electron scattering data by means of extraction of a distorted spectral function (depends on two variables)