Covariant density functional theory: Inclusive charged current neutrino-nucleus reactions.

Lądek Zdrój, Febr. 9, 2009

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N. Paar, D. Vretenar, T.Marketin, P. R. PRC 77, 24608 (2008)

Content

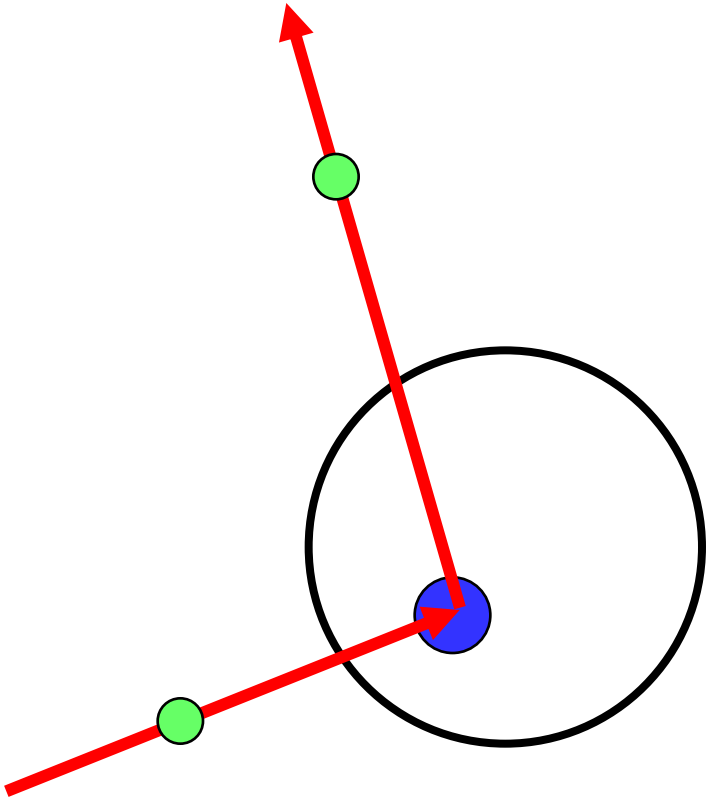
- **neutrino-nucleus charge current reactions**

- **Covariant density functional theory**
  - relativistic Hartree Bolgoliubov for the groundstate
  - relativistic QRPA for excited state
  - relativistic pnQRPA for IAR and GTR

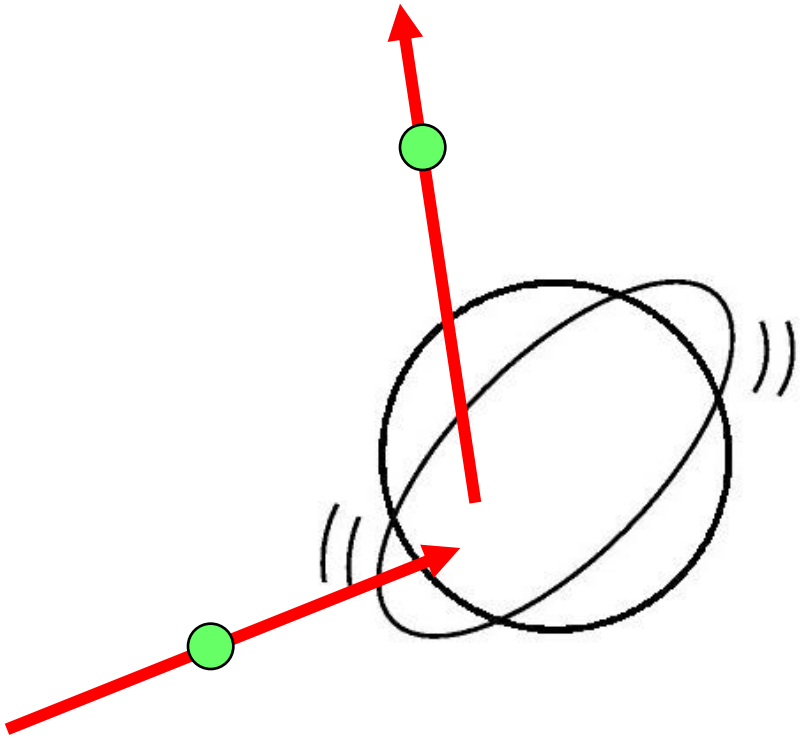
- **Applications: neutrino-nucleus cross sections**
  - neutrino-nucleus cross section on C, O, Ar, Fe, Pb,
  - allowed and forbidden transitions
  - cross section averaged over neutrino flux
  - comparison with experiments

- **Conclusions**

Response of the nucleus on an incoming particle

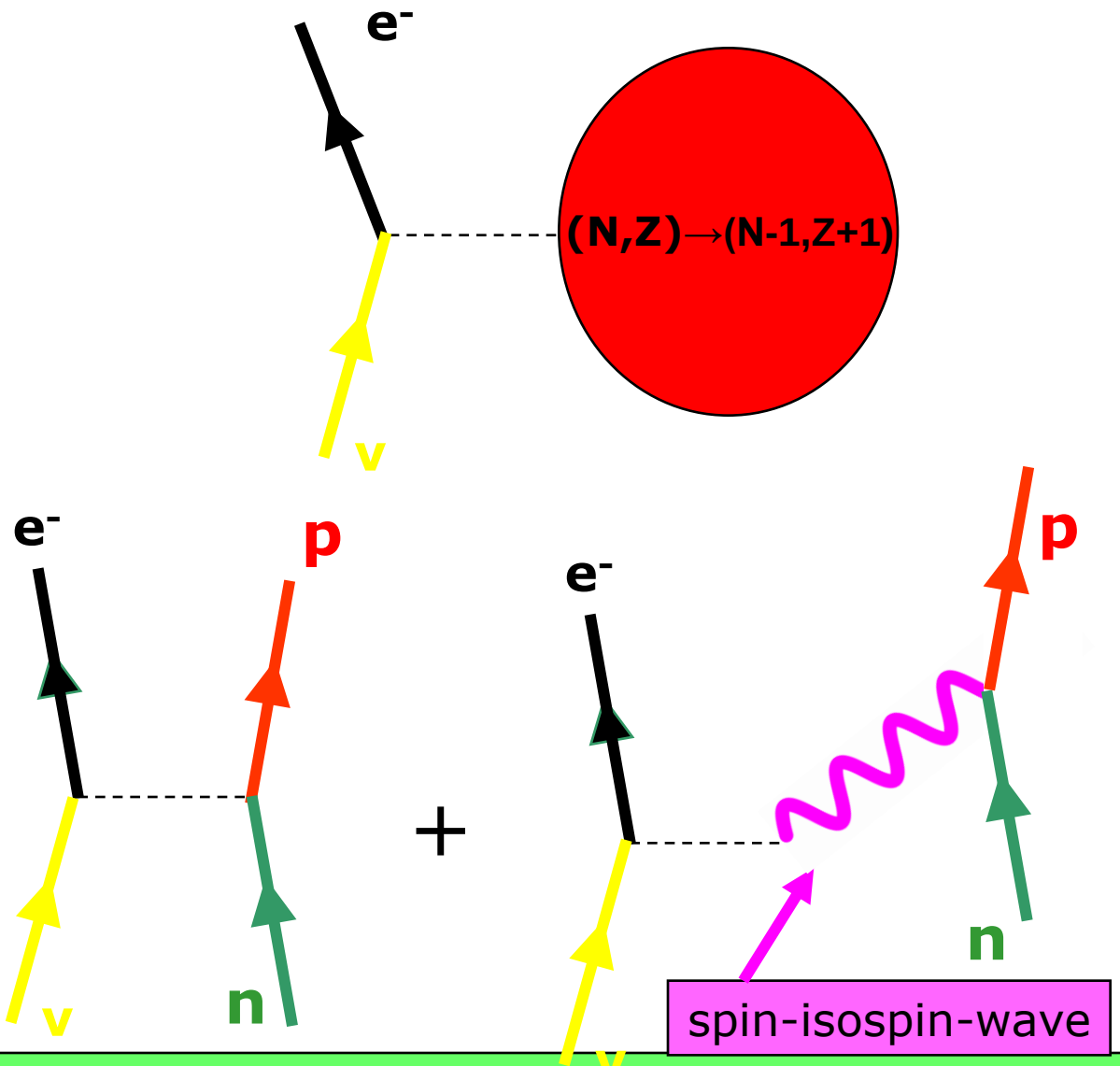


Scattering on a single nucleon



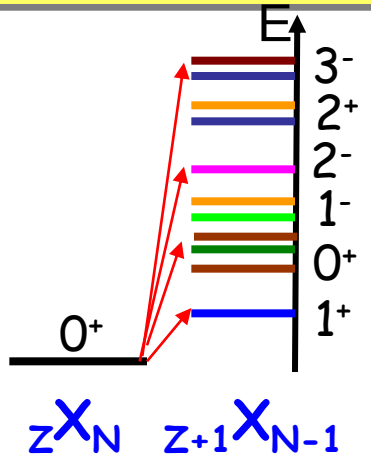
excitation of the whole nucleus

neutrino scattering



neutrino-nucleus charged current reactions

$$\nu_e + {}_Z X_N \rightarrow {}_{Z+1} X^*_{N-1} + e^-$$



The main challenges:

1. To improve our understanding of relevant excitations involved
2. Detector response for neutrino experiments
3. Neutrino nucleo-synthesis → production of heavier elements
4. Low-energy beta beams for ν-nucleus interaction studies

Available experimental data are rather limited → deuteron, ^{12}C, ^{56}Fe

neutrino-nucleus cross section:

$$\left(\frac{d\sigma_\nu}{d\Omega}\right) = \frac{1}{(2\pi)^2} V^2 k_l E_l \sum_{l-spins} \frac{1}{2J_i+1} \sum_{M_i M_f} |\langle f|\hat{H}_W|i\rangle|^2$$

Hamiltonian for the weak interaction

$$\hat{H}_W = -\frac{G}{\sqrt{2}} \int d\boldsymbol{x} \mathcal{J}^\lambda(\boldsymbol{x}) j_\lambda(\boldsymbol{x})$$

transition nuclear matrix elements
(assuming plane waves for leptons)

$$\langle f|\hat{H}_W|i\rangle = -\frac{G}{\sqrt{2}} l_\lambda \int d\boldsymbol{x} e^{-i\boldsymbol{q}\boldsymbol{x}} \langle f|\mathcal{J}^\lambda(\boldsymbol{x})|i\rangle$$

Extreme relativistic limit : final lepton energy $E_l >> m_l c^2$

Walecka, Donnely

$$\left(\frac{d\sigma_\nu}{d\Omega}\right)_{ERL} = \frac{2G_F^2 cos^2\theta_c}{\pi}\frac{E_l^2}{2J_i+1}$$

$$\times \left\{\left(\frac{q_\lambda^2}{2q^2}cos^2\frac{\theta}{2} + sin^2\frac{\theta}{2}\right)\sum_{J\geq 1}\left[|\langle J_f||\hat{T}_J^{MAG}||J_i\rangle|^2 + |\langle J_f||\hat{T}_J^{EL}||J_i\rangle|^2\right]\right.$$

$$-sin\frac{\theta}{2}\sqrt{\frac{q_\lambda^2}{q^2}cos^2\frac{\theta}{2} + sin^2\frac{\theta}{2}}\sum_{J\geq 0} 2Re\langle J_f||\hat{T}_J^{MAG}||J_i\rangle\langle J_f||\hat{T}_J^{EL}||J_i\rangle^*$$

$$\left.+cos^2\frac{\theta}{2}\sum_{J\geq 0}|\langle J_f||\hat{\mathcal{M}}_J - \frac{q_0}{|q|}\hat{\mathcal{L}}_J||J_i\rangle|^2\right\}$$

Coulomb & longitudinal

multipole operators

Transverse magnetic & Transverse electric multipole operators

neutrino-nucleus reactions in open shell nuclei.

Reduced matrix elements for various transition operators include the details of the nuclear structure
(nuclear ground state and excitations)

$$\langle J_f || \hat{\mathcal{O}}^J || J_i \rangle$$

Recent microscopic studies improved significantly description of weak interaction rates, however, there are several problems:

1. SHELL MODEL → accurate in description of the ground state wave functions, description of high-lying states necessitates a large model space which is problematic to treat numerically

Different interactions in various mass regions employed, only lower mass nuclei can be studied, problems in convergence with the model space

2. HYBRID MODEL

$$\sigma_\nu^{tot}(E_\nu) = \sigma_\nu^{SM}(E_\nu) + \sigma_\nu^{RPA}(E_\nu)$$

SHELL MODEL – Strasbourg-Madrid codes by Caurier et al.
→Gamow-Teller strength (with quenching factor)
(+ rescaled by a factor to account finite momentum transfer)

Occupation numbers

CONTINUUM RPA → forbidden transitions

GROUND STATE:

Woods-Saxon potential fitted to single-particle energies

→

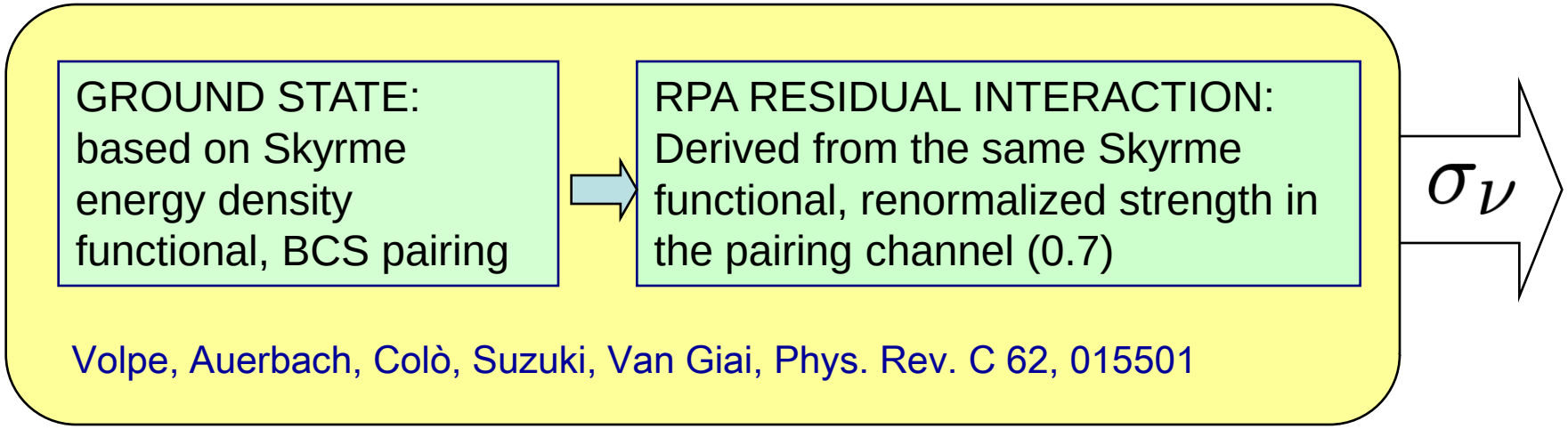
RPA RESIDUAL INTERACTION:
G-matrix from Bonn potential or zero-range Landau Migdal force

+ adjustment factor to rescale the weak interaction rates

σ_ν

Kolbe, Langanke, Martinez-Pinedo, Phys. Rev. C 60, 052801 (1999).

3. Skyrme Hartree-Fock + Quasiparticle RPA → towards a consistent description of weak interaction rates along the nuclide chart



Different transition operators are employed in different approaches, (e.g. expansion in momentum transfer, only Gamow-Teller, first forbidden transitions,…) → it is difficult to compare results

NEW → Neutrino-nucleus cross sections based on self-consistent Relativistic Quasiparticle Random Phase Approximation (RQRPA)

Relativistic Hartree-Bogoliubov model (RHB)
+ Relativistic quasiparticle RPA (RQRPA)

Both the field for the ground state and the residual QRPA interaction is derived from the same effective Lagrangian

Pairing correlations are described by the pairing part of the finite range Gogny interaction D1S

σ_ν

reduced matrix elements for the neutrino-nucleus cross section

$$\langle J_f||\hat{\mathcal{O}}^J||J_i\rangle = \sum_{pn}\langle p||\hat{\mathcal{O}}^J||n\rangle\left(X_{pn}^{fJ}u_p v_n + (-1)^J Y_{pn}^{fJ} v_p u_n\right)$$

RQRPA amplitudes

RHB occupation probabilities

Density functional theory in nuclei

$$E = \langle \Psi | \hat{H} | \Psi \rangle = \langle \Phi | \hat{H}_{eff} | \Phi \rangle = E[\hat{\rho}]$$

$|\Phi\rangle$ **Slater determinant** $\Leftrightarrow$ $\hat{\rho}$ **density matrix**

$$|\Phi\rangle = \mathcal{A}(\varphi_1(\mathbf{r}_1) \cdots \varphi_A(\mathbf{r}_A)) \qquad \hat{\rho}(\mathbf{r},\mathbf{r}') = \sum_{i=1}^{A} |\varphi_i(\mathbf{r})\rangle\langle\varphi_i(\mathbf{r}')|$$

Mean field:

$$\hat{h} = \frac{\delta E}{\delta \hat{\rho}}$$

Eigenfunctions:

$$\hat{h}|\varphi_i\rangle = \varepsilon_i |\varphi_i\rangle$$

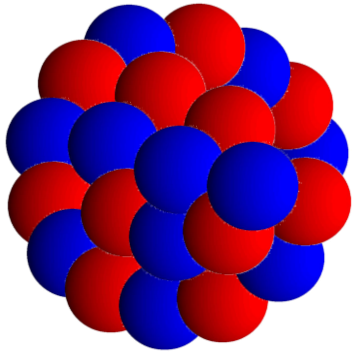
Interaction:

$$\hat{V} = \frac{\delta^2 E}{\delta\hat{\rho}\delta\hat{\rho}}$$

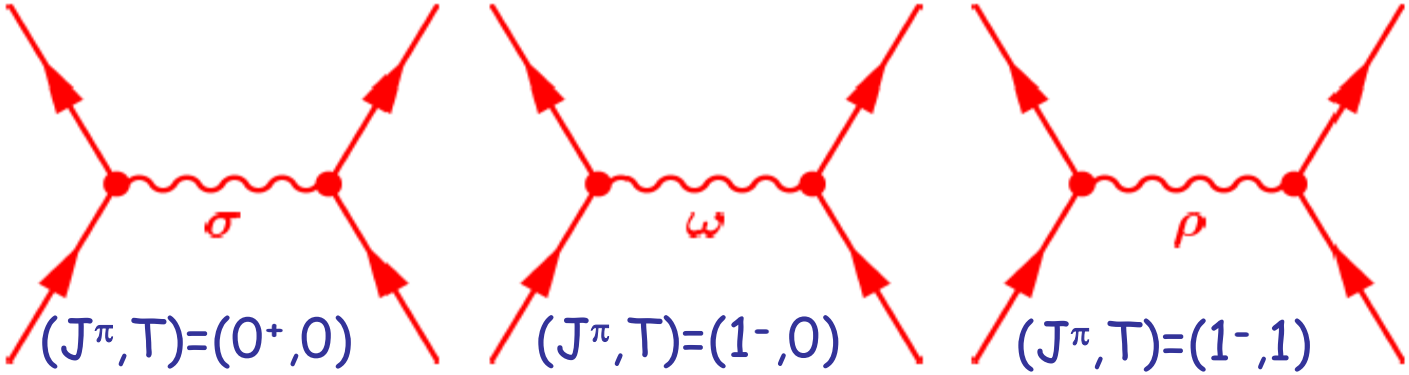
Extensions: Pairing correlations, Covariance
Relativistic Hartree Bogoliubov (RHB)

Walecka model

$E[\hat{\rho}]$



Nucleons are coupled by exchange of mesons through an effective Lagrangian (EFT)



$$S(\mathbf{r}) = g_\sigma \sigma(\mathbf{r}) \qquad V(\mathbf{r}) = g_\omega \omega(\mathbf{r}) + g_\rho \vec{\tau}\vec{\rho}(\mathbf{r}) + eA(\mathbf{r})$$

Sigma-meson: attractive scalar field

Omega-meson: short-range repulsive

Rho-meson: isovector field

Three relativistic models:

Meson exchange with non-linear meson couplings:

Boguta and Bodmer, NPA. 431, 3408 (1977)
Lalazissis, J. König, P.R., PRC 55. 540 (1997)

$$\frac{1}{2}m_\sigma^2\sigma^2 \quad \Rightarrow \quad U(\sigma)=\frac{1}{2}m_\sigma^2\sigma^2+\frac{1}{3}g_2\sigma^3+\frac{1}{4}g_3\sigma^4$$

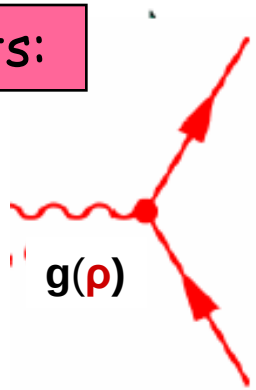
NL1, NL3, TM1, ..

Meson exchange with density dependent coupling constants:

R.Brockmann and H.Toki, PRL 68, 3408 (1992)
Lalazissis, Niksic, Vretenar, P.R., PRC 71, 024312 (2005)

DD-ME1, DD-ME2

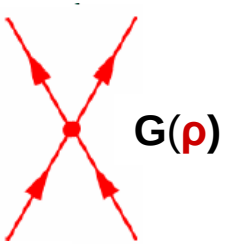
8 parameters

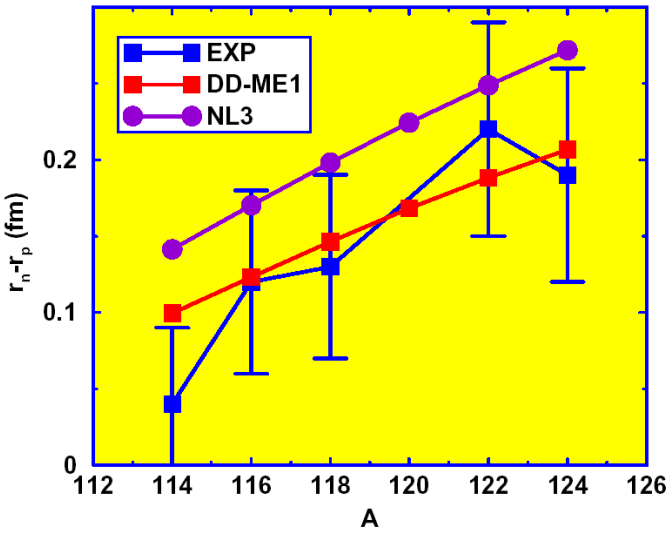


Point-coupling models with density dependent coupling constants:

Manakos and Mannel, Z.Phys. **330**, 223 (1988)
Buervenich, Madland, Maruhn, Reinhard, PRC 65, 44308 (2002)

PC-F1, ...





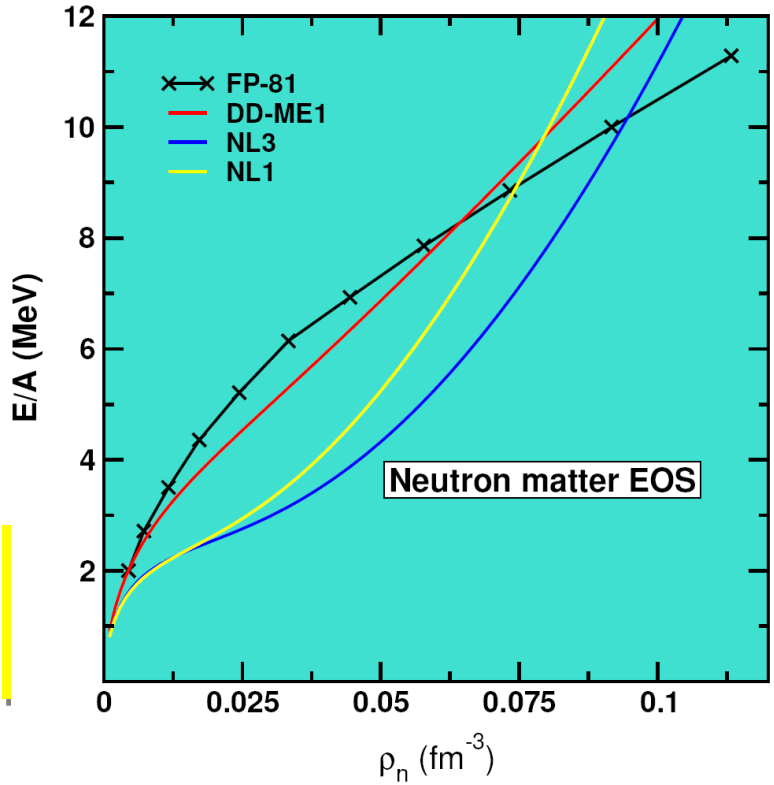
$$\alpha \equiv \frac{N-Z}{N+Z}$$

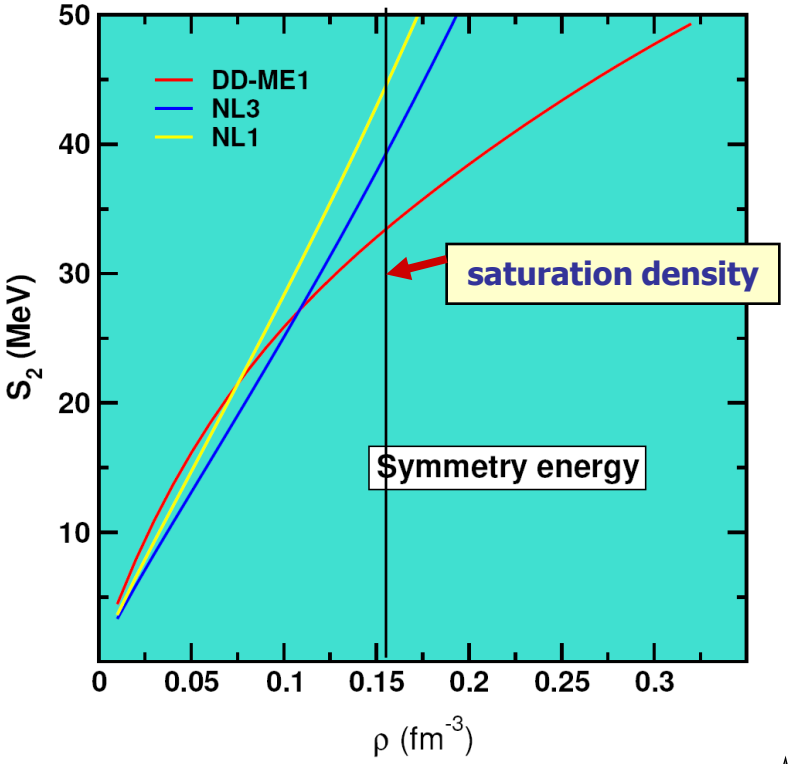
$$E(\rho,\alpha) = E(\rho,0) + S_2(\rho)\alpha^2 + S_4(\rho)\alpha^4 + \cdots$$

$$S_2(\rho) = a_4 + \frac{p_0}{\rho_{sat}^2}(\rho-\rho_{sat}) + \frac{\Delta K_0}{18\rho_{sat}^2}(\rho-\rho_{sat})^2 + \cdots$$

empirical values:

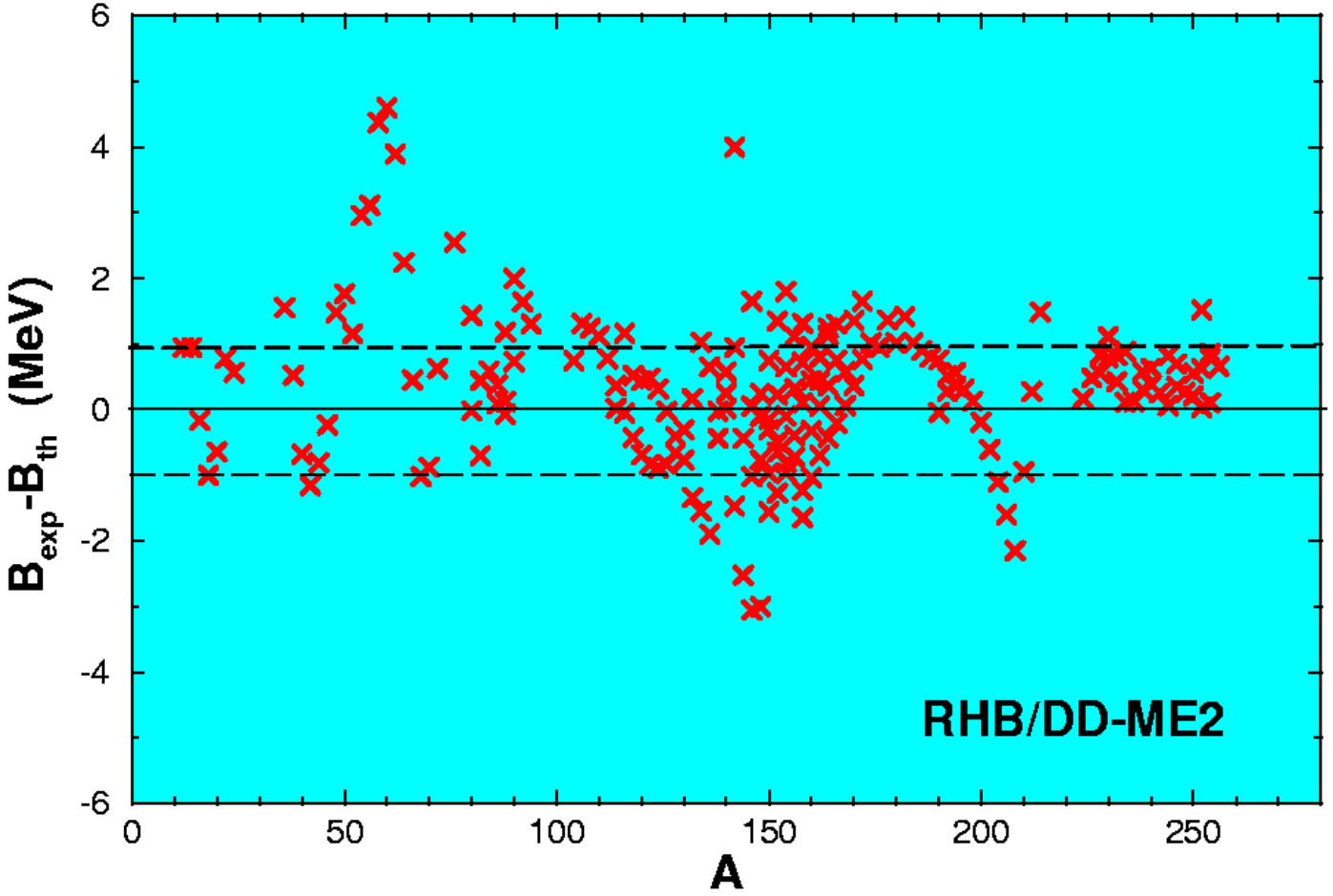
30 MeV ≤ a_4 ≤ 34 MeV
2 MeV/fm³ < p_0 < 4 MeV/fm³
-200 MeV < ΔK_0 < -50 MeV





rms-deviations: masses: $\Delta m = 900$ keV
radii: $\Delta r = 0.015$ fm

Lalazissis, Niksic, Vretenar, Ring, PRC 71, 024312 (2005)



Relativistic RPA for excited states

Small amplitude limit:

$$\hat{\rho}(t) = \hat{\rho}^{(0)} + \delta\hat{\rho}(t)$$

ground-state density

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar\omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

$\delta\rho_{ph}$, $\delta\rho_{\alpha h}$

$\delta\rho_{hp}$, $\delta\rho_{h\alpha}$

RRPA matrices:

$$A_{minj} = (\epsilon_n - \epsilon_i)\delta_{mn}\delta_{ij} + \frac{\partial h_{mi}}{\partial \rho_{nj}}, \qquad B_{minj} = \frac{\partial h_{mi}}{\partial \rho_{jn}}$$

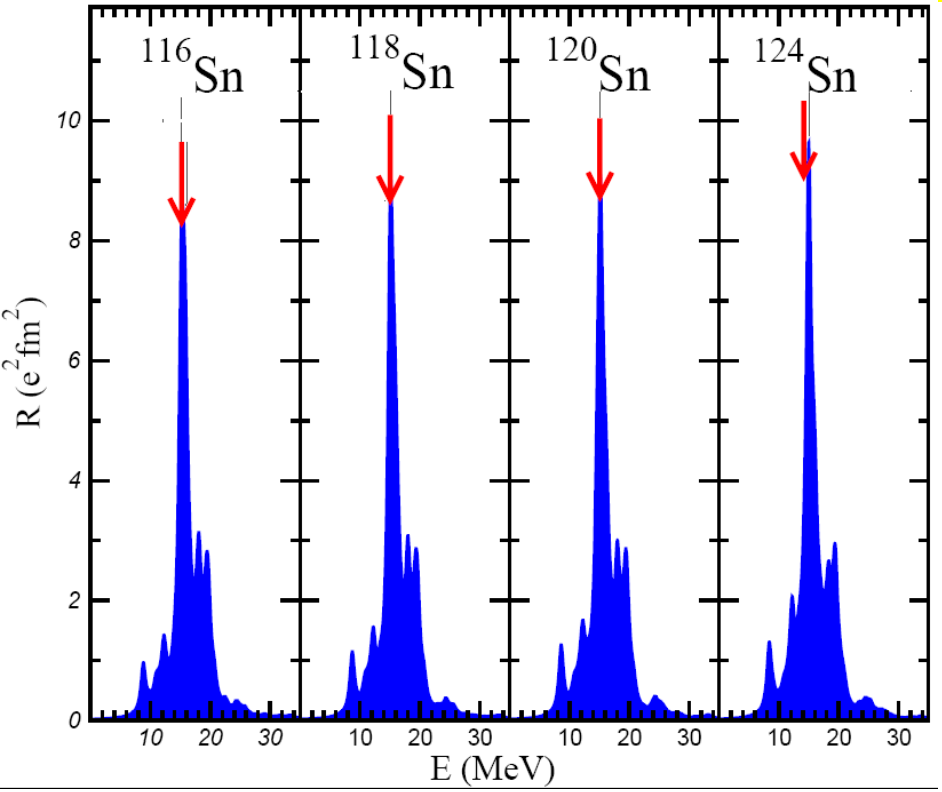
⇒ the same effective interaction determines the Dirac-Hartree single-particle spectrum and the residual interaction

⇒ pairing force is Gogny-D1S

Interaction:

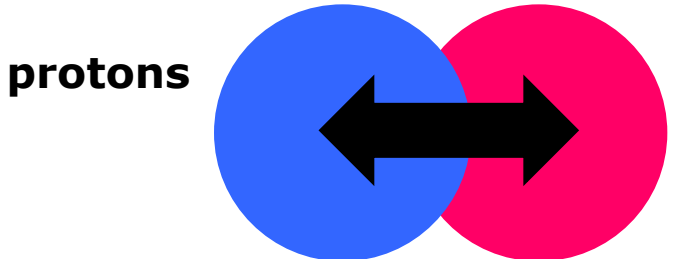
$$\hat{V} = \frac{\delta^2 E}{\delta\,\hat{\rho}\,\delta\,\hat{\rho}}$$

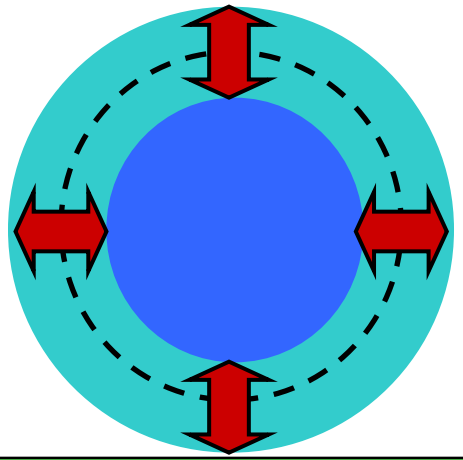
Relativistic (Q)RPA calculations of giant resonances



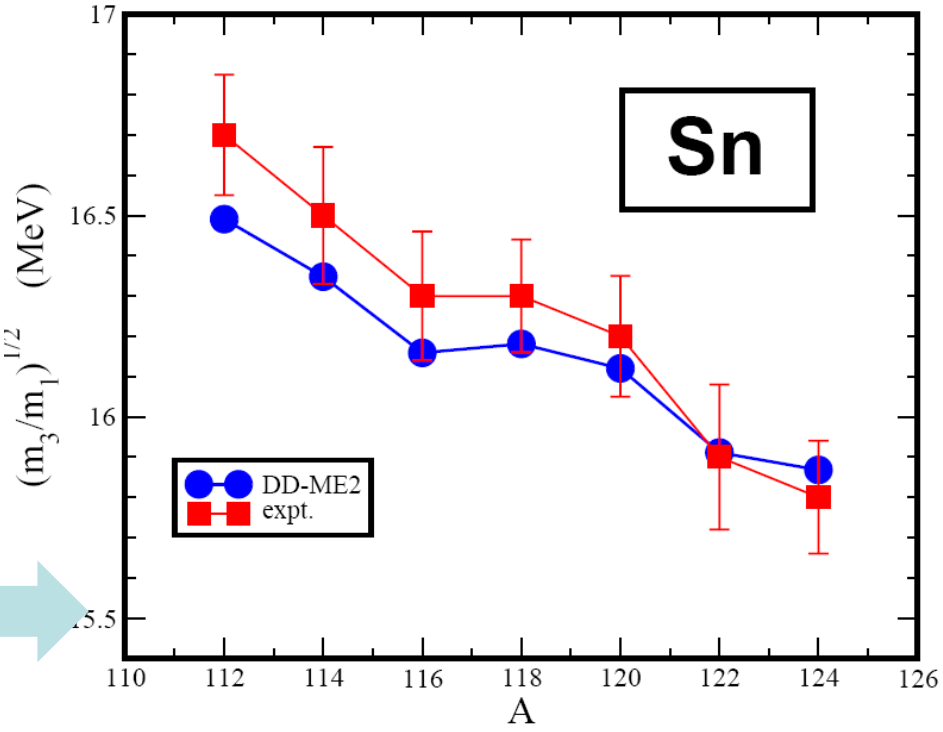
Sn isotopes: DD-ME2 effective interaction + Gogny pairing

Isovector dipole response





Isoscalar monopole response



Spin-Isospin Resonances: IAR - GTR

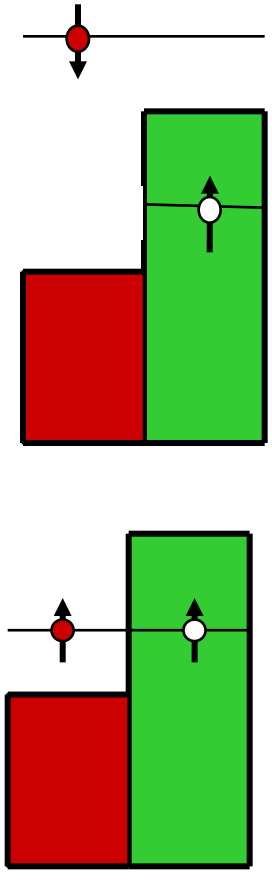
Z,N **Z+1,N-1**

$$|GTR\rangle = S_- T_+ |Z,N\rangle$$

spin flip **σ**

$$|Z,N\rangle \longrightarrow |IAR\rangle = T_+ |Z,N\rangle$$

isospin flip **τ**



$$E_{GTR} - E_{IAR} \sim \Delta(l \cdot s) \sim \frac{dV}{dr} \sim \text{neutron skin} = r_n - r_p$$

Spin-Isospin Resonances: IAS and GTR

charge-exchange excitations → **proton-neutron relativistic QRPA**

π and ρ-meson exchange generate the spin-isospin dependent interaction terms

$$\mathcal{L}_{\pi N} = -\frac{f_\pi}{m_\pi}\bar{\psi}\gamma_5\gamma_\mu\partial^\mu\vec{\pi}\vec{\tau}\psi$$

the Landau-Migdal zero-range force in the spin-isospin channel

$$V(1,2) = g'_0\left(\frac{f_\pi}{m_\pi}\right)^2 \vec{\tau}_1\cdot\vec{\tau}_2\ \boldsymbol{\Sigma}_1\cdot\boldsymbol{\Sigma}_2\ \delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$$

(g'_0=0.55)

GAMOW-TELLER RESONANCE: S=1 T=1 $J^\pi = 1^+$

ISOBARIC ANALOG STATE: S=0 T=1 $J^\pi = 0^+$

Isobaric Analog Resonance: IAR

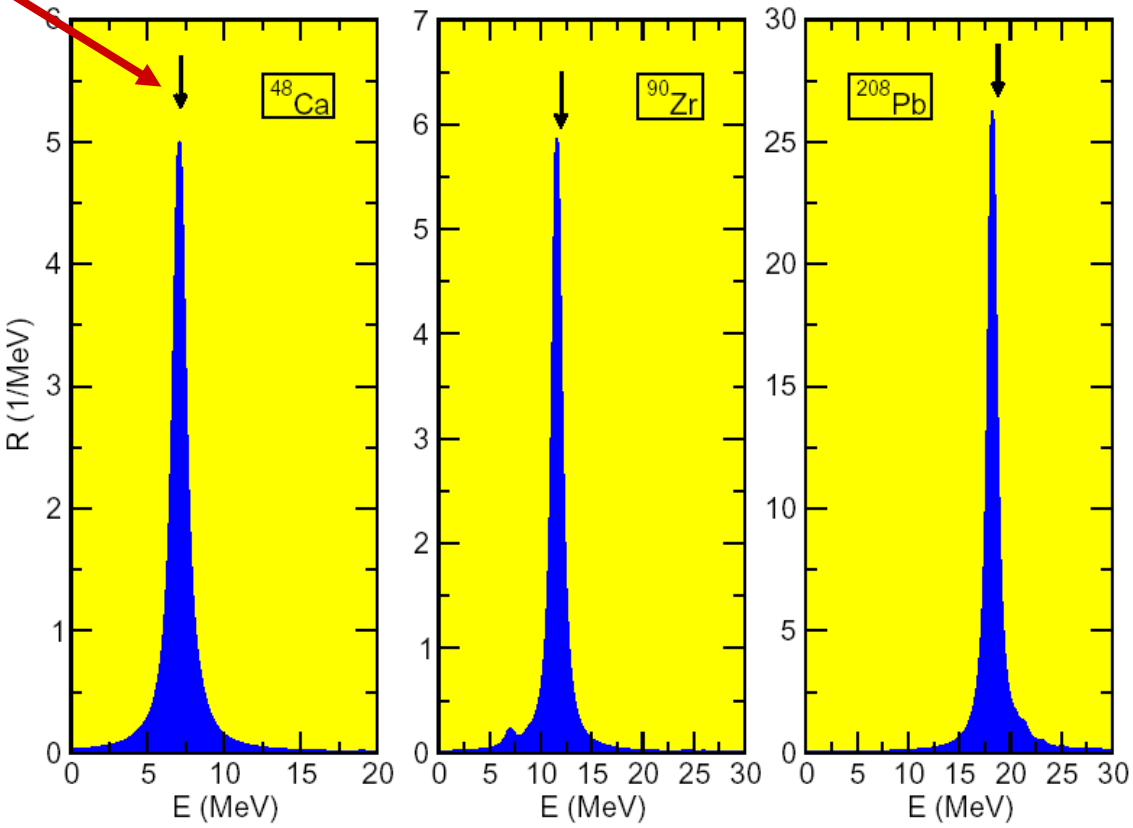
N. Paar, T. Niksic, D. Vretenar, P.Ring, PR C69, 054303 (2004)

experiment

Isospin-flip excitations

$$T^{F}_{\beta^{\pm}} = \sum_{i=1}^{A} \tau_{\pm}$$

S=0 T=1 $J^{\pi} = 0^{+}$



GT-Resonances

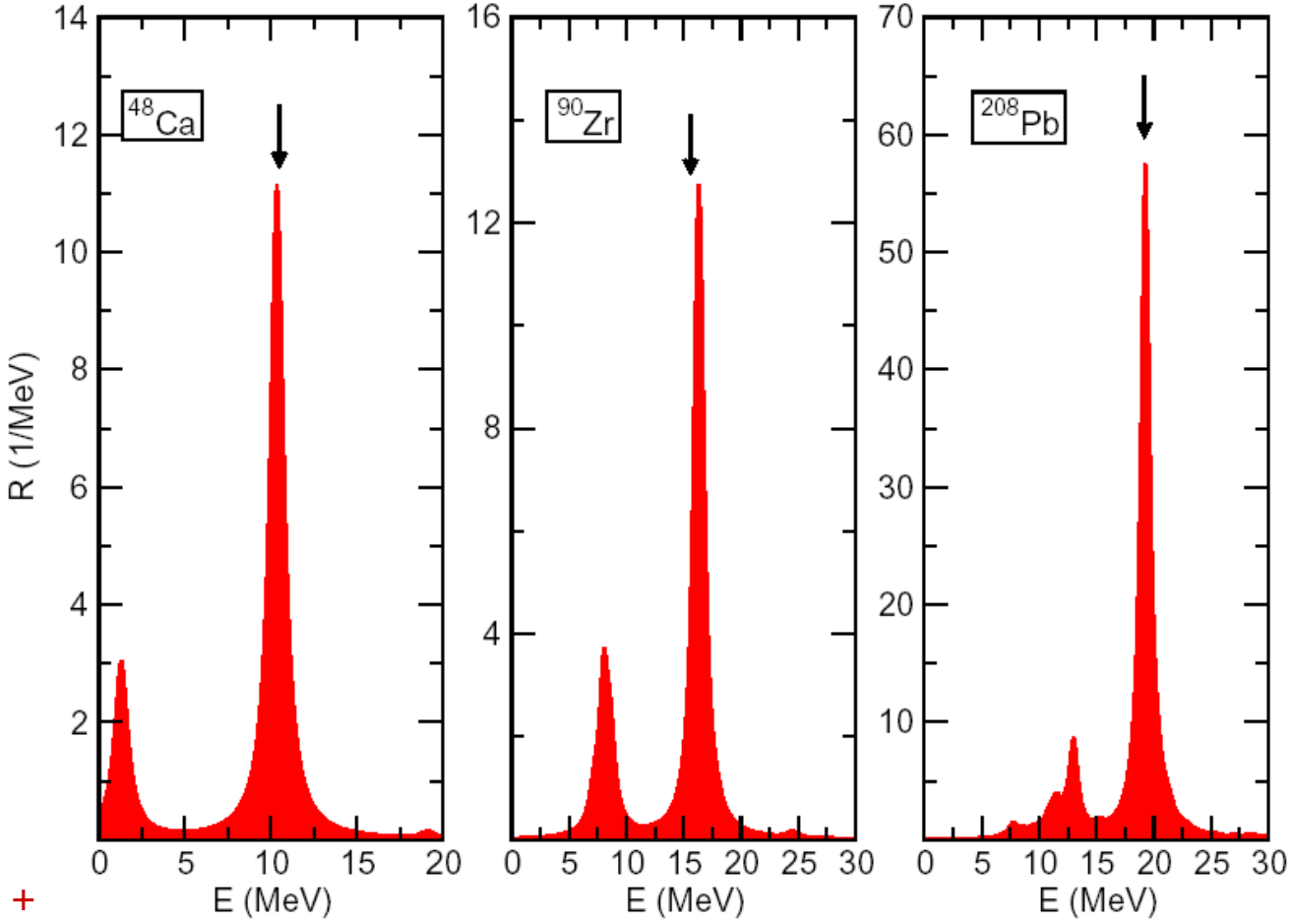
N. Paar, T. Niksic, D. Vretenar, P.Ring, PR C69, 054303 (2004)

Spin-flip

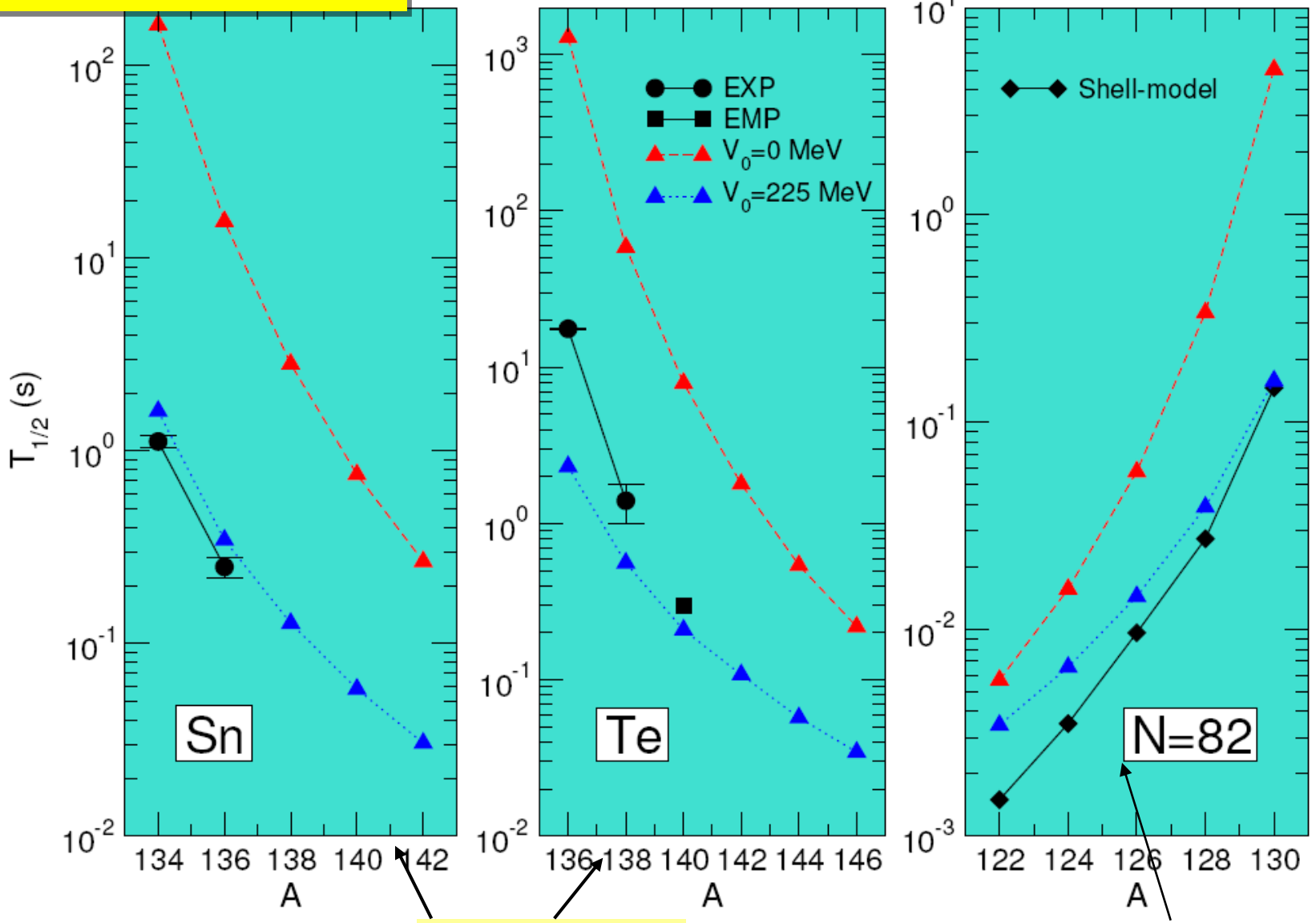
isospin-flip excitations

$$T^{GT}_{\beta^{\pm}} = \sum_{i=1}^{A} \Sigma\tau_{\pm}$$

S=1 T=1 $J^{\pi} = 1^{+}$



β-decay



T. Niksic et al, PRC 71, 014308 (2005)

$\nu h_{9/2} -> \pi h_{11/2}$

G. Martinez-Pinedo and K. Langanke, PRL 83, 4502 (1999)

neutrino nucleus charged current cross section

- relativistic Hartree-Bogoliubov calculation
  for the ground state: N=20 shells, DD-ME2, Gogny D1S

- relativistic pn-QPRA calculation
  with the same interaction in the canonical basis,
  fully selfconsistent

- calculation of the various matrix elements

Coulomb correction due to effect of nuclear charge on outgoing lepton

Fermi function

$$F(Z,E_l) = F_0(Z,E_l)L_0$$

$$F_0(Z,E_l) = 4(2p_lR)^{2(\gamma-1)}\left|\frac{\Gamma(\gamma+iy)}{\Gamma(2\gamma+1)}\right|^2 e^{\pi y}$$

$$\left\{\begin{array}{l} \gamma = \sqrt{1-(\alpha Z)^2} \\ y = \alpha Z\dfrac{E_l}{p_l} \end{array}\right.$$

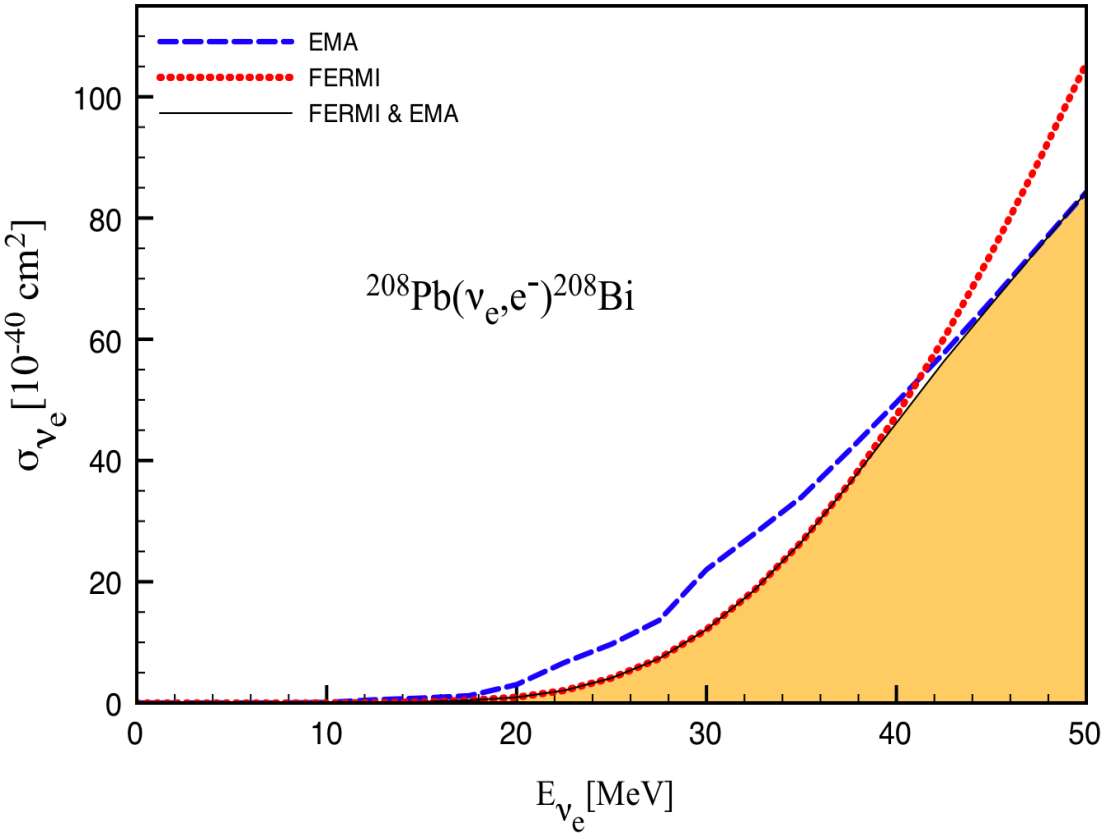
Effective momentum approximation:

$$p_l^{eff} = \sqrt{E_l^{eff}-m_e^2}$$

$$E_l^{eff} = E_l - V_C(0)$$

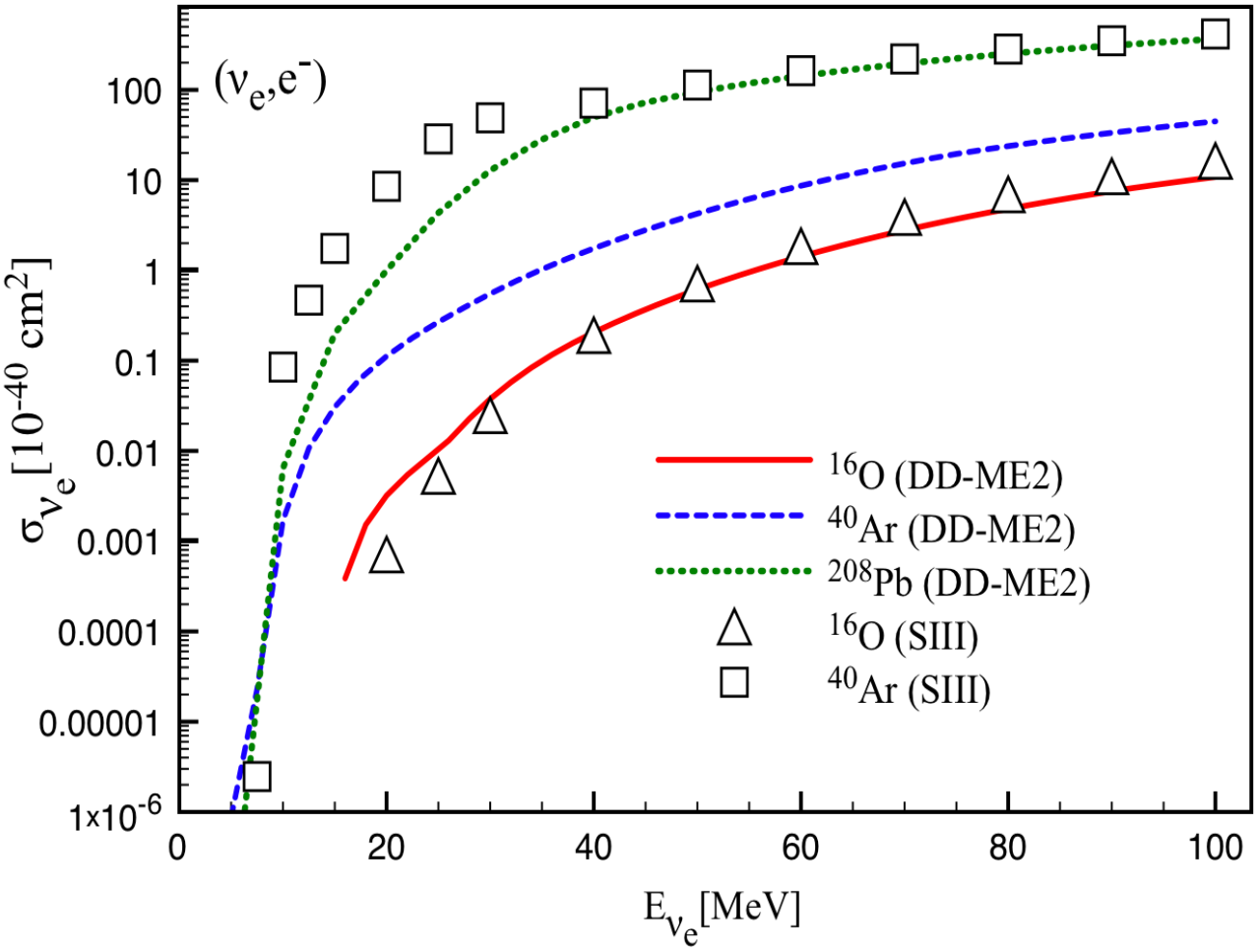
$$p_lE_l \rightarrow p_l^{eff}E_l^{eff}$$

Engel, Phys. Rev. C 57, 2004 (1998)

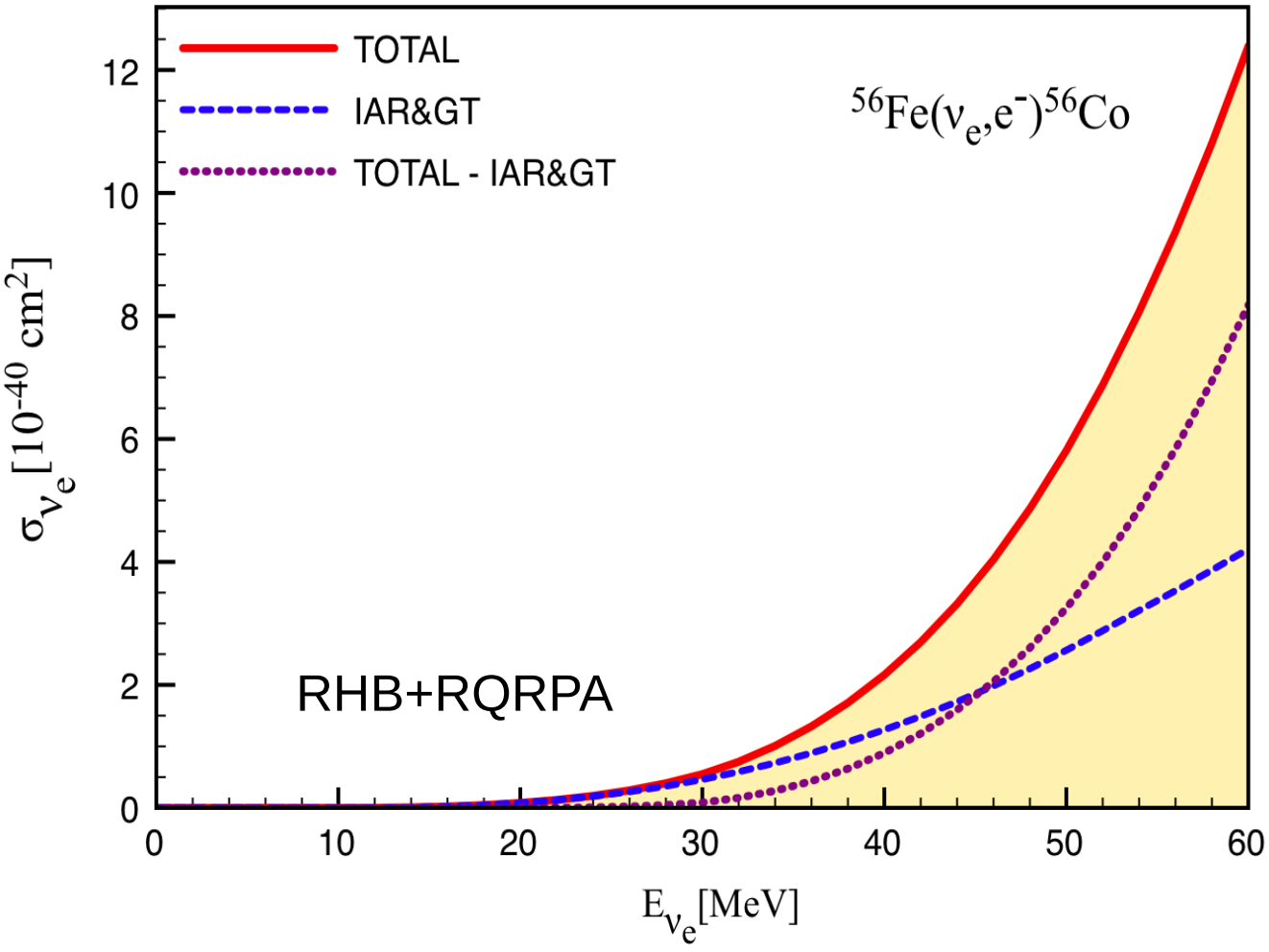


RHB+RQRPA neutrino-nucleus cross section

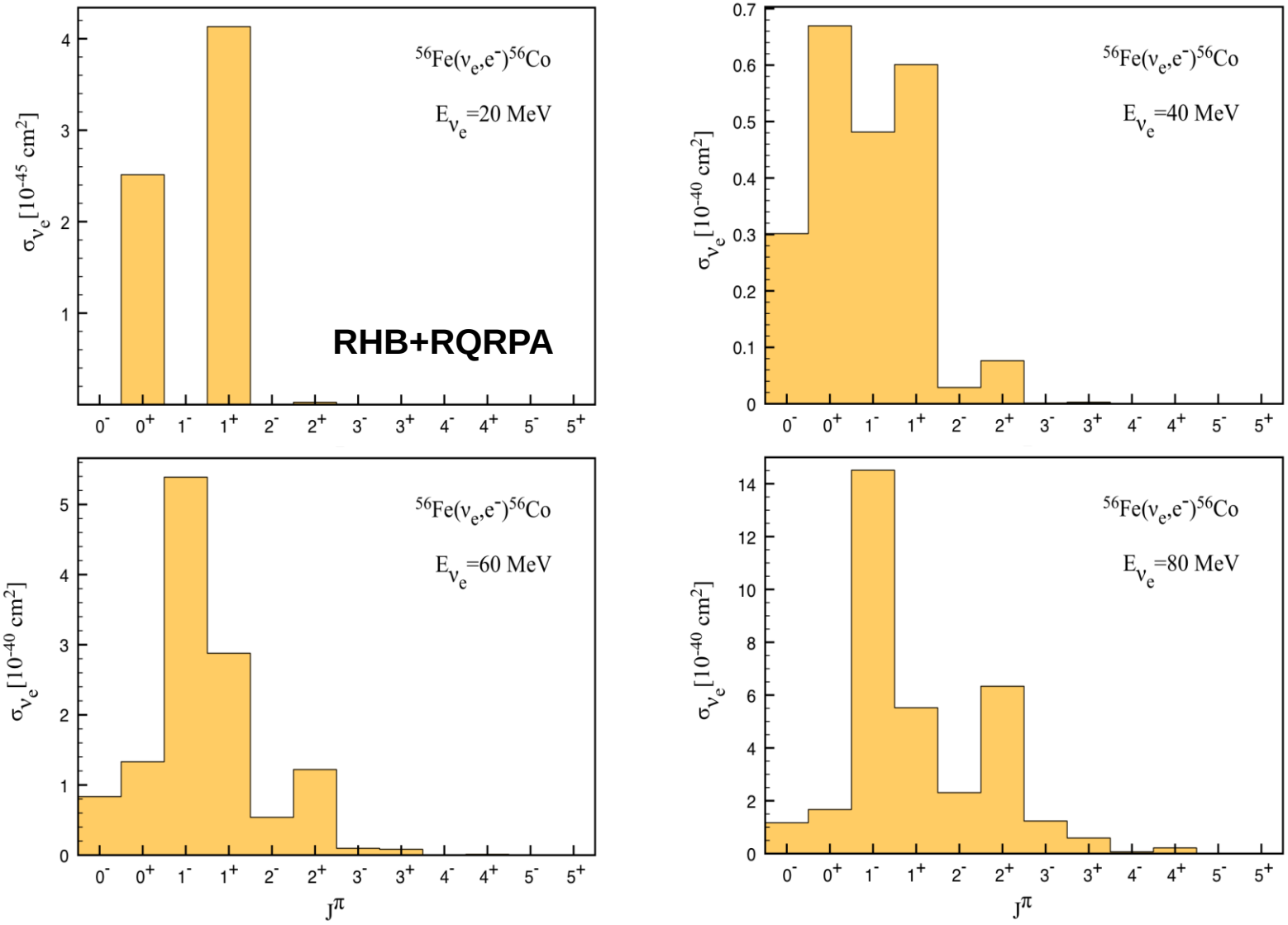
comparison with SIII (Lazauskas + Volpe, 2007):



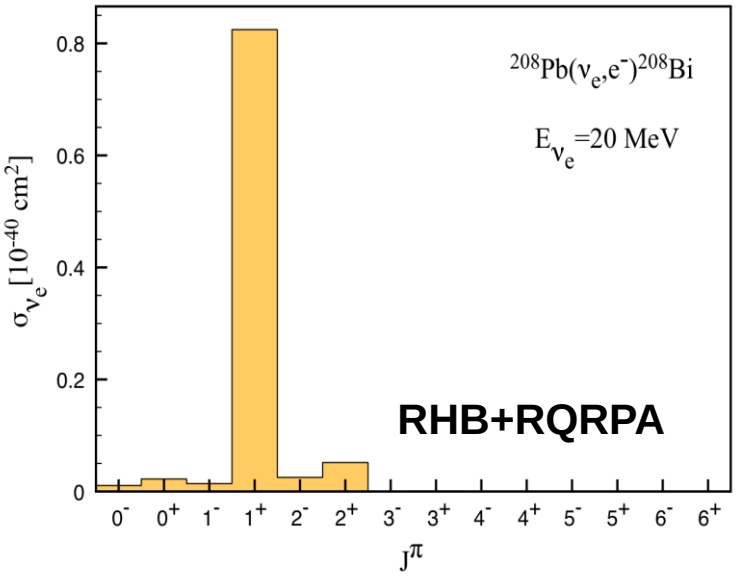
RHB-RQRPA neutrino-nucleus ^{56}Fe cross section

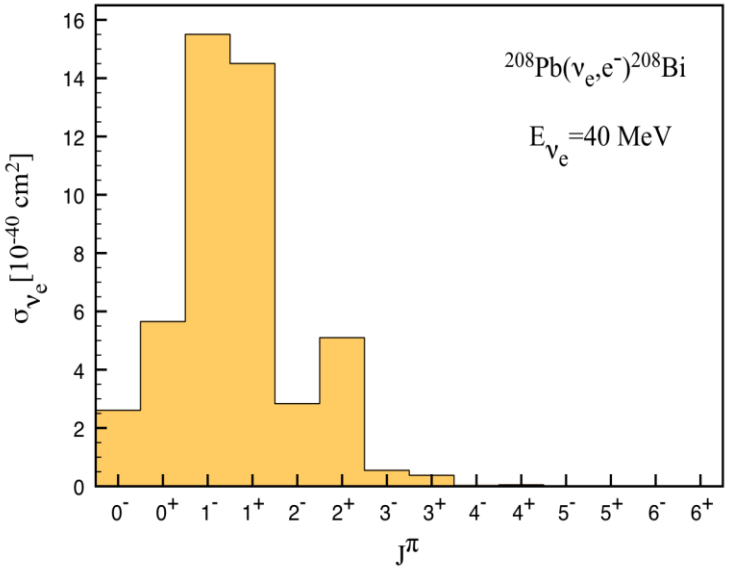


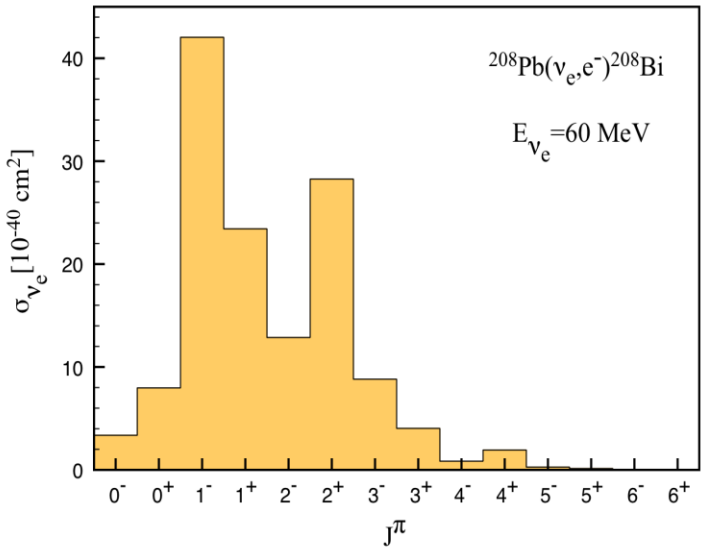
Distribution of cross section over multipolarities



Distribution of cross section over multipolarities

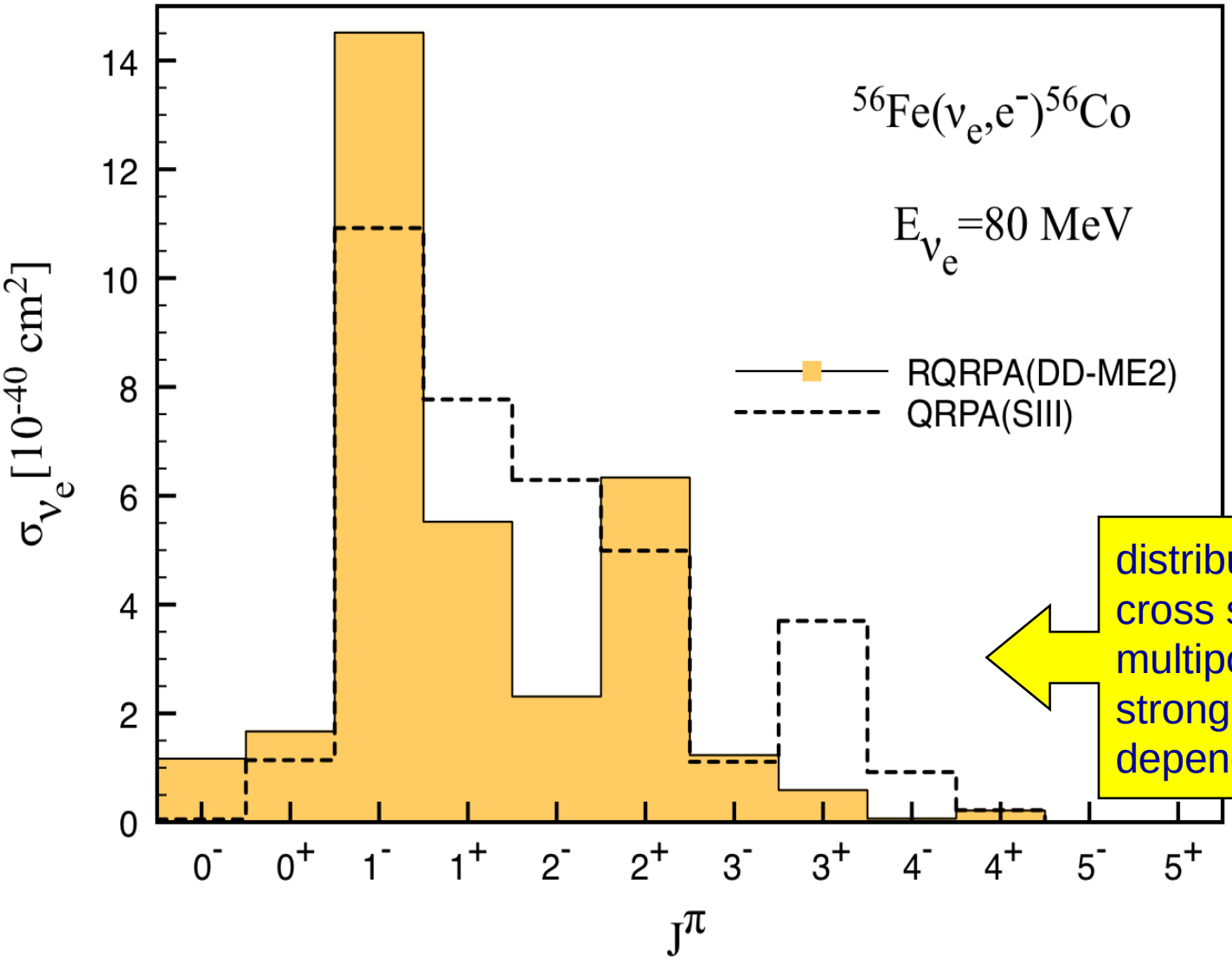






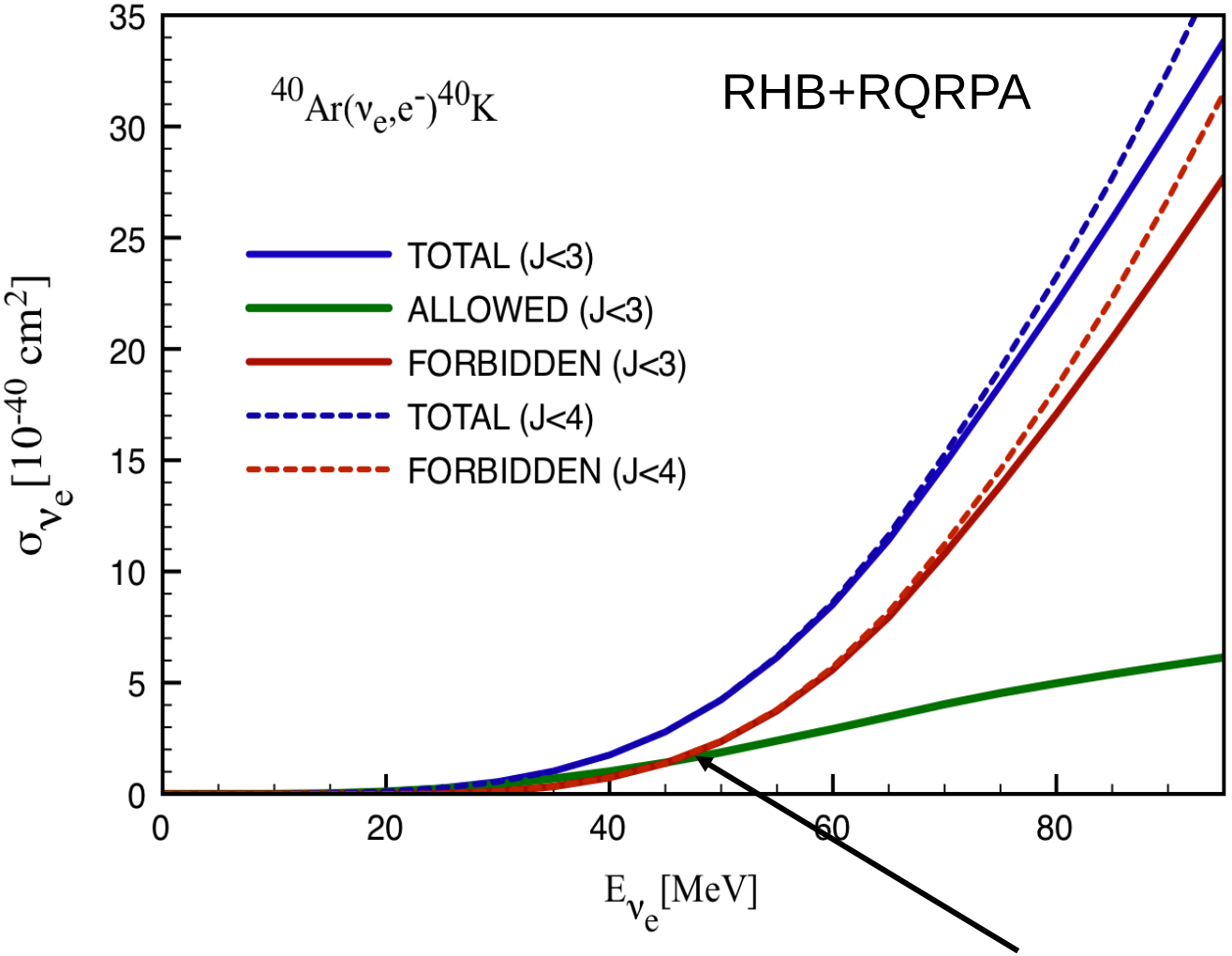
The knowledge on spin-dipole excitations and higher multipolarities is essential for reliable description of neutrino nucleus cross sections !

Distribution of cross section over multipolarities



distribution of cross sections over multipolarities is strongly model dependent

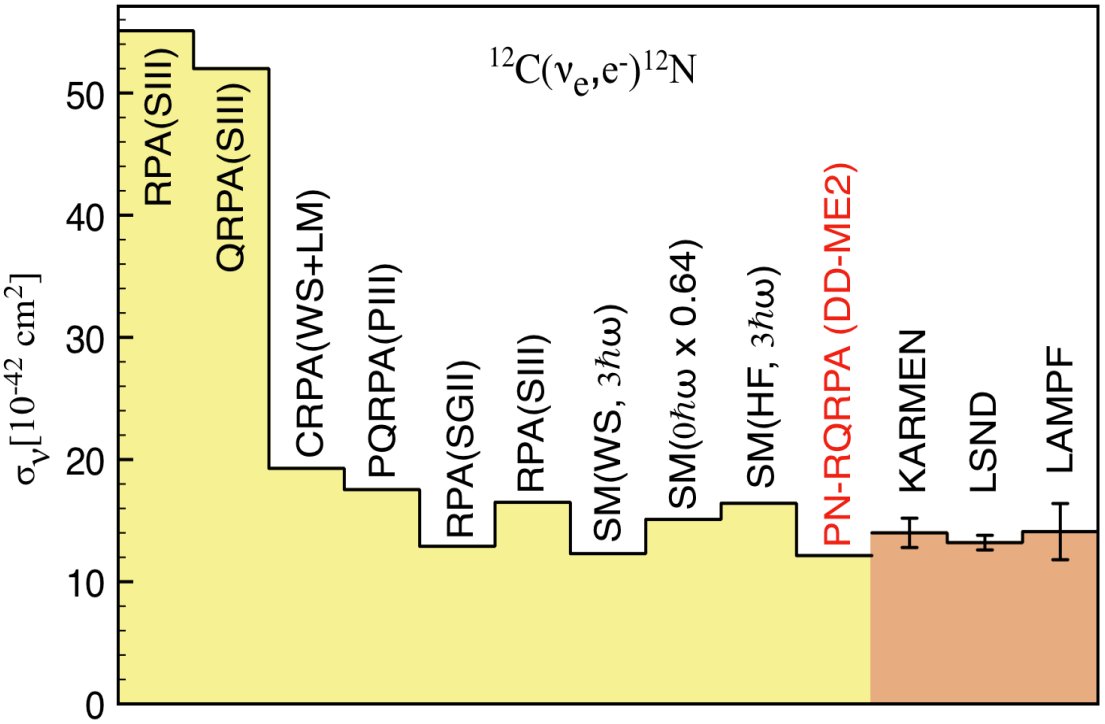
Contributions from allowed and forbidden transitions

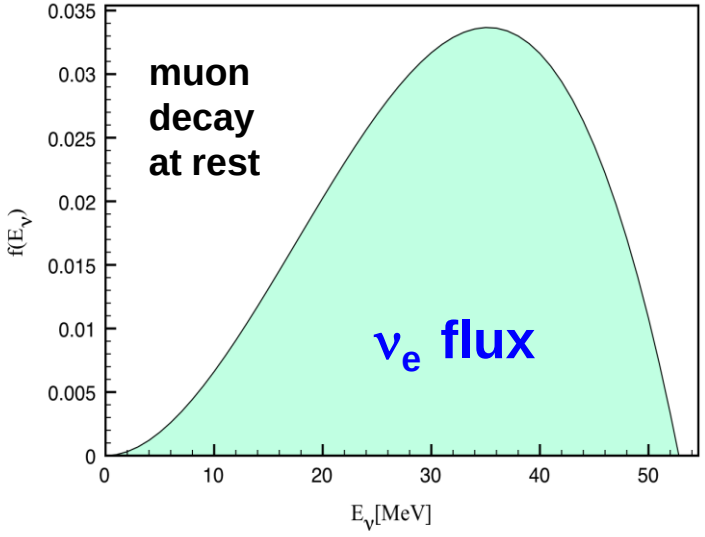


Crossing between the cross section with allowed and forbidden transisions !

Cross section (ν_e,e^-) averaged over supernova neutrino flux

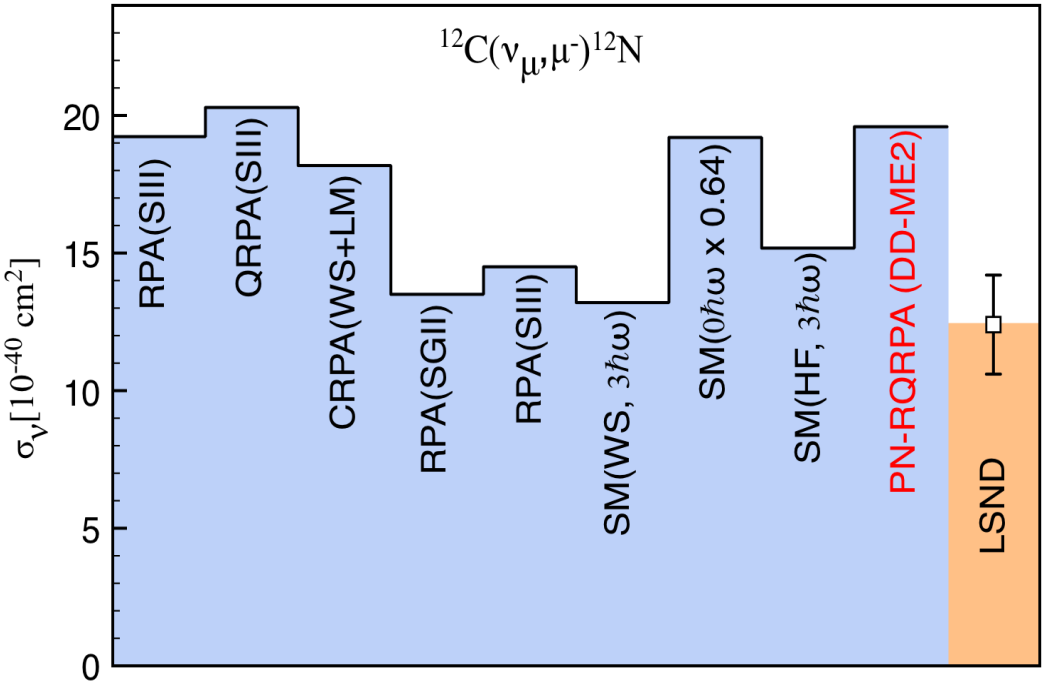
$$\langle \sigma_\nu \rangle = \frac{\int dE_\nu \sigma_\nu(E_\nu) f(E_\nu)}{\int dE'_\nu f(E'_\nu)}$$

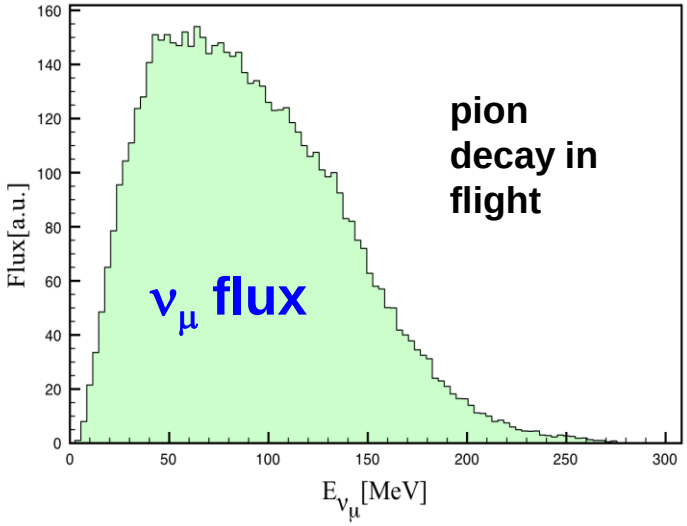




Cross section (ν_μ,μ^-) averaged over supernova neutrino flux

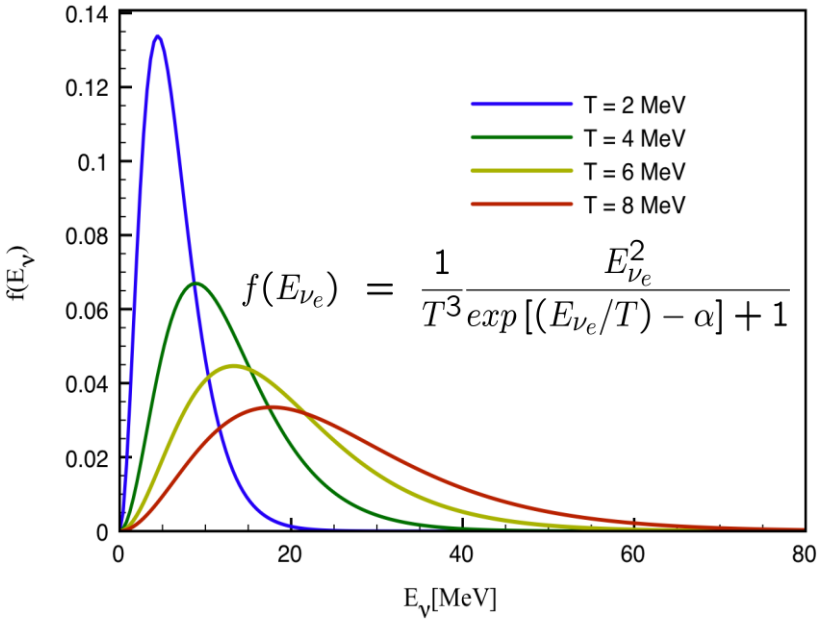
$$\langle \sigma_\nu \rangle = \frac{\int dE_\nu \sigma_\nu(E_\nu) f(E_\nu)}{\int dE'_\nu f(E'_\nu)}$$

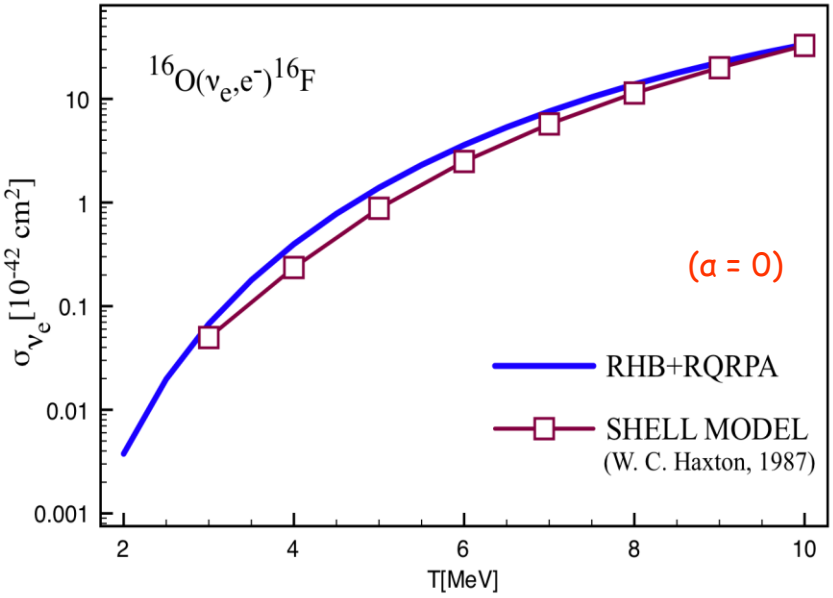


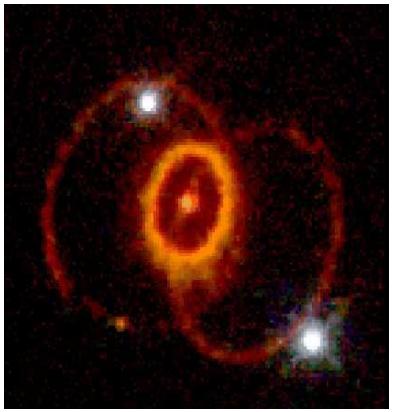


Cross section averaged over supernova neutrino flux

Supernova neutrino flux is
given by Fermi-Dirac spectrum

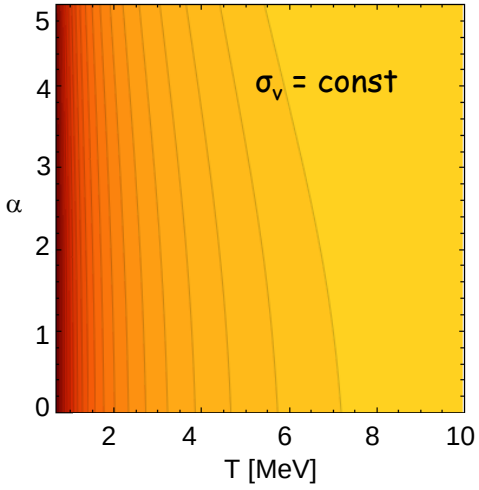






Cross section averaged over
Supernova neutrino flux

$$\langle \sigma_\nu \rangle = \frac{\int dE_\nu \sigma_\nu(E_\nu) f(E_\nu)}{\int dE'_\nu f(E'_\nu)}$$



CONCLUDING REMARKS

Relativistic QRPA framework provides consistent description of nuclear modes of excitation and neutrino-nucleus charged current reactions

Better knowledge on spin-dipole and excitations of higher multipolarities can improve description of neutrino-nucleus reactions

→ Valuable and sensitive tests for the many body theory and effective interactions employed

→ Neutrino-nucleus cross sections for neutrino detectors for beta-beams …

Consistent description of neutrino-nucleus reactions at the limits of stability:

Relevance for nuclear astrophysics

→ Neutrino nucleosynthesis

→ nuclear modes