C5A axial form factor determined from the bubble chamber experiments

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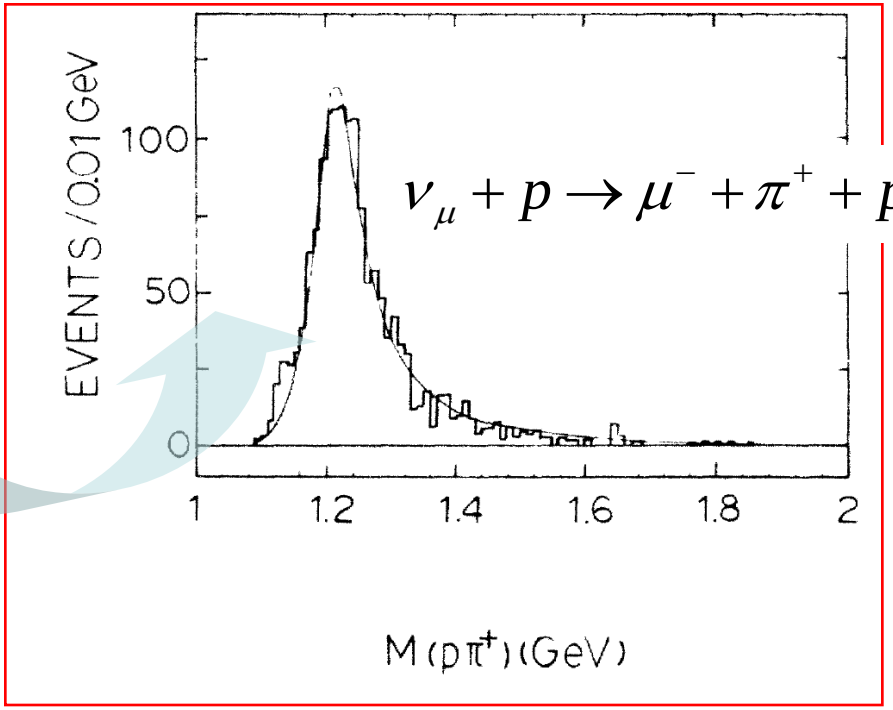
A common project with J. Sobczyk, D. Kiełczewska and P. Przewłocki

Motivations

- Δ(1232) excitation induced by ν-nucleon interaction
- → Simultaneous analysis of the data from two experiments: ANL and BNL
- Extraction of the axial contribution from bubble chamber experiments
  - **Fits either to ANL or to BNL data**
- Input to →**NuWro** Monte Carlo Generator
- New fits of cross sections and C5A with account of their uncertainties
- → application to 1π0 production in NC ν-nucleon scattering

Nonresonant background negligible



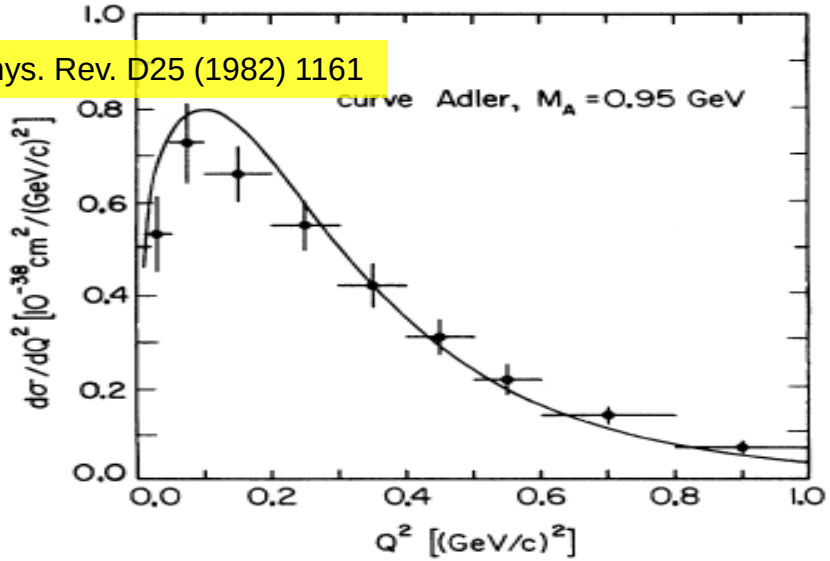
T. Kitagaki *et al.* Phys. Rev. D 34 (1986) 2554

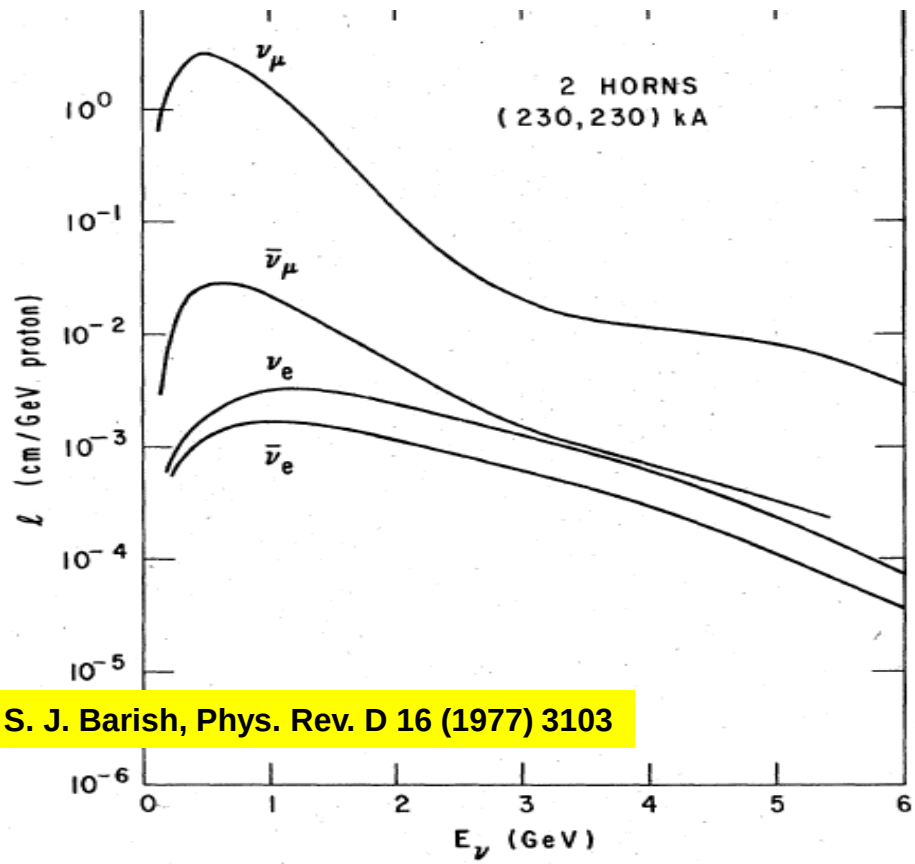
12 ft @ ANL

- 12 foot bubble chamber filled with **deuterium** and hydrogen @ Argonne National Laboratory
- **S. J. Barish, Phys. Rev. D19 (1979) 2521.**
- **G. M. Radecky Phys. Rev. D25 (1982) 1161.**
- <E> < 1 GeV
- <Δflux> =15% (E<1.5 GeV) and 25% (above)
- Differential cross sections in Q2
- Total cross sections
- Kinematical cuts:
  - 0.5 GeV < E < 6.0 GeV
  - 0.01 GeV2 < Q2 < 1 GeV2
  - W < 1.4 GeV

$$\nu + d \rightarrow \mu^- + \Delta^{++}(1232) + n$$

G. M. Radecky Phys. Rev. D25 (1982) 1161



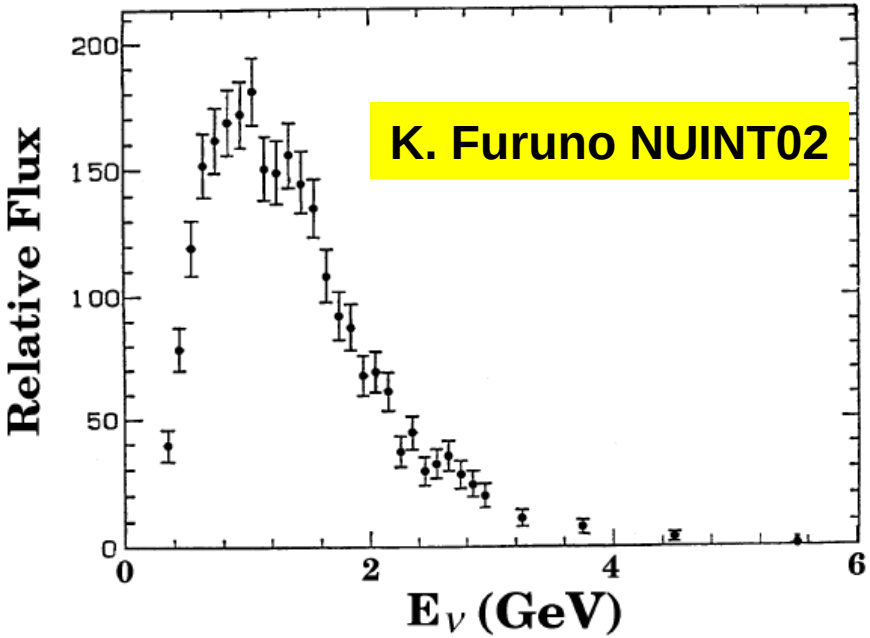


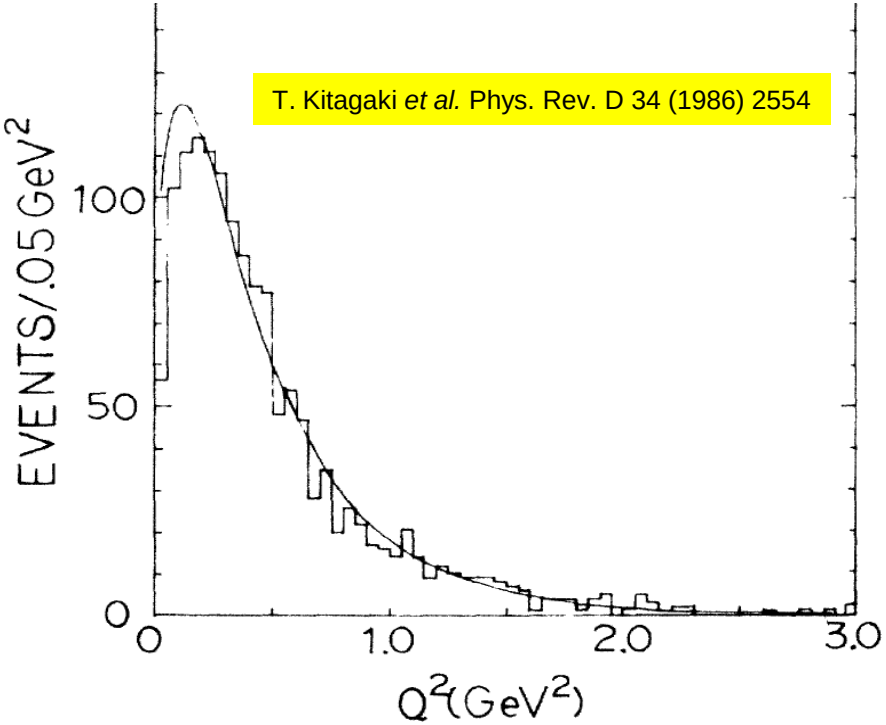
S. J. Barish, Phys. Rev. D 16 (1977) 3103

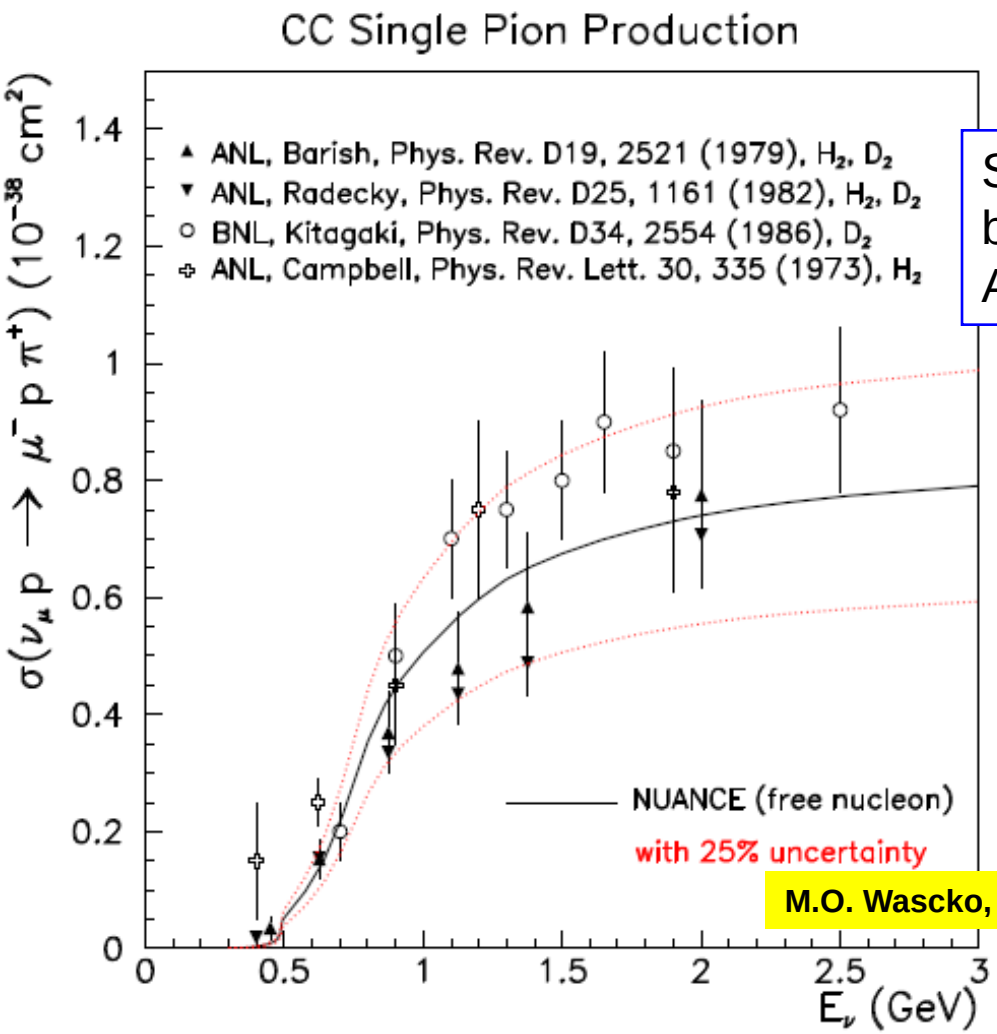
7-ft @BNL

$$\nu + d \rightarrow \mu^- + \Delta^{++}(1232) + n$$

- 7 foot bubble chamber filled with **deuterium** at Brookhaven National Laboratory.
- <E> = 1.6 GeV
- **T. Kitagaki *et al.* Phys. Rev. D 34 (1986) 2554.**
- **T. Kitagaki *et al.*, Phys. Rev. D42 (1990) 1331.**
- <Δflux> = 10%
- Kinematical cuts:
  - 0.5 GeV < E < 6.0 GeV
  - Q2 < 3 GeV2 but:
    - Q2 > 0.1 → efficiency!
  - W<1.4 GeV
- Total cross sections
- Normalized cross sections







Systematic shift (around 20%) between total
ANL and BNL cross sections?

M.O. Wascko, Nucl. Phys. Proc. Suppl. 159 (2006) 50

Figure 1. *Previous measurements of the total cross section per nucleon of the process* $\nu_\mu p \rightarrow \mu^- p \pi^+$ *at low neutrino energy.*

Rarita-Schwinger Formalism

$$\nu + p \rightarrow \mu^- + \Delta^{++}(1232)$$

$$\mathcal{J}_\mu^{CC} = \mathcal{J}_\mu^V + \mathcal{J}_\mu^A;$$ **Hadronic current**

The differential cross section for Δ production is:

$$\frac{d^2\sigma}{dW dQ^2} = \frac{\widetilde{G}^2 W}{64\pi^2 M E^2} L^{\mu\nu} W_{\mu\nu},$$

where $\widetilde{G} = G\cos\theta_c$, and G is the Fermi constant, and θ_c Cabibbo angle.

The hadronic tensor is:

$$W_{\mu\nu} = \frac{1}{4MM_\Delta}\frac{1}{2}\sum_{spin}\langle\Delta^{++}, p'|\mathcal{J}_\mu^{CC}|p\rangle\langle\Delta^{++}, p'|\mathcal{J}_\nu^{CC}|p\rangle^* \frac{\Gamma_\Delta/2}{((W-M_\Delta)^2+\Gamma_\Delta^2/4)}$$

The leptonic tensor is:

$$\mathrm{L}_{\mu\nu} = 8\left(k'_\mu k_\nu + k_\mu k'_\nu - g_{\mu\nu}k'_\alpha k^\alpha \mp i\epsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta\right).$$

$$\langle \Delta^{++}(p')| \mathcal{J}^V_\mu |N(p)\rangle = \sqrt{3}\bar{\Psi}_\lambda(p') \left[g^\lambda{}_\mu \left(\frac{C_3^V}{M}\gamma_\nu + \frac{C_4^V}{M^2}p'_\nu + \frac{C_5^V}{M^2}p_\nu \right) q^\nu - q^\lambda \left(\frac{C_3^V}{M}\gamma_\mu + \frac{C_4^V}{M^2}p'_\mu + \frac{C_5^V}{M^2}p_\mu \right) \right] \gamma_5 u(p)$$

$$C_5^V(Q^2) = 0, \quad C_4^V(Q^2) = -\frac{M}{W}C_3^V(Q^2)$$

SU(6) symmetry relation

→We apply fits of CiV proposed by O. Lalakulich *et al.* **Phys. Rev. D 74 (2006) 014009**

$$\langle \Delta^{++}(p')| \mathcal{J}^A_\mu |N(p)\rangle = \sqrt{3}\bar{\Psi}_\lambda(p') \left[g^\lambda{}_\mu \left(\gamma_\nu \frac{C_3^A}{M} + \frac{C_4^A}{M^2}p'_\nu \right) q^\nu - q^\lambda \left(\frac{C_3^A}{M}\gamma_\mu + \frac{C_4^A}{M^2}p'_\mu \right) + g^\lambda{}_\mu C_5^A + \frac{q^\lambda q_\mu}{M^2}C_6^A \right] u(p)$$

Adler model predictions (dispertion theory):
C3A = 0, and:

$$C_4^A(Q^2) = -C_5^A(Q^2)/4$$

PCAC

$$C_6^A(Q^2) = \frac{M^2}{m_\pi^2 + Q^2}C_5^A(Q^2)$$

P. A. Schreiner and F. Von Hippel, Nucl. Phys. B58 (1973) 333

$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$$

Dipole parameterization

From Adler model:
a=-1.21, b= 2 GeV2

Adler parameterization

$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{aQ^2}{b+Q^2}\right)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$$

P. A. Schreiner and F. Von Hippel, Nucl. Phys. B 58 (1973) 333.

$$C_5^A(0) = \frac{g_{\pi N\Delta} f_\pi}{\sqrt{6}M} \approx 1.15 \pm 0.01$$

From PCAC
see: D. Barquilla-Cano, *et al.* Phys. Rev. C75 (2007) 065203 [Erratum-ibid. C 77 (2008) 019903]

other parameterizations!

$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{Q^2}{M_{A1}^2}\right)^{-2}\left(1+\frac{Q^2}{M_{A2}^2}\right)^{-i=1,2}$$

Adler with a, and b

Parameters seem to be correlated

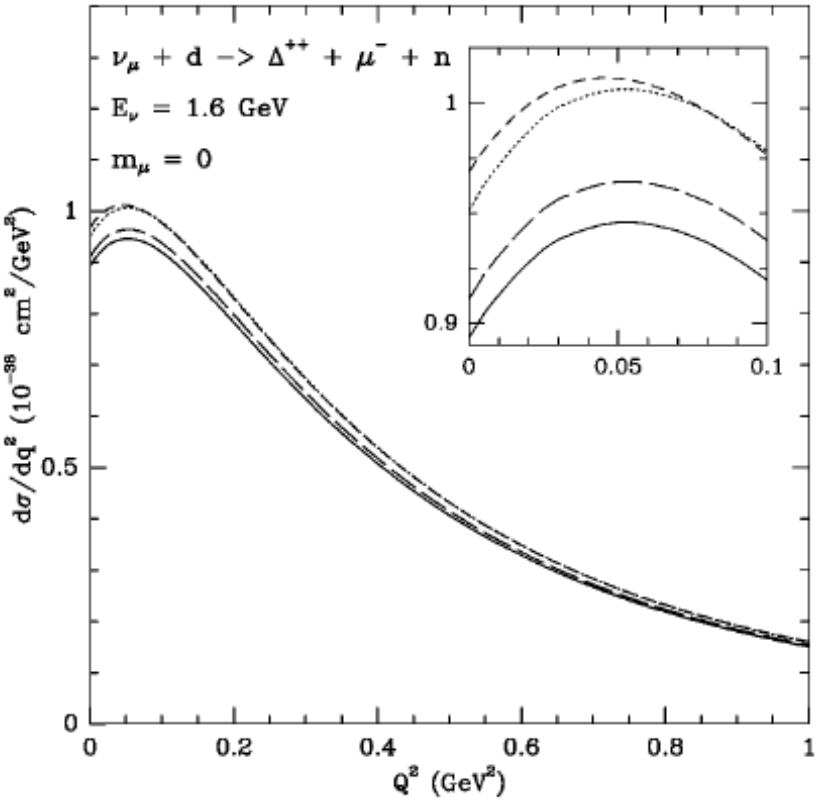


FIG. 1. Differential cross section for weak charged current neutrino production of Δ on deuteron. In the short-dashed line, deuteron effects are neglected while dotted, long-dashed, and solid lines include these effect using Hulthen, Bonn and Paris deuteron wave functions, respectively.

L.Alvarez-Ruso, S.K.Singh and M.J.Vicente Vacas, Phys. Rev. C 59 (1999) 3386

- **NN potentials**
  - Hulthen, ***L. Hulthen and M. Sugawara, Handbuch der Physik***
  - Bonn, ***R. Machleidt, K. Holinde and C. Elster, Phys. Rept. 149 (1987) 1***
  - Paris: ***M. Lacombe, B. Loiseau, R. Vinh Mau, J. Cote, P. Pires and R. de Tourreil, Phys. Lett. B 101 (1981) 139***

Used in this analysis

$$R_{ANL}(Q^2) = \frac{\left(d\sigma(\nu d \to \mu^- n \Delta^{++})/dQ^2\right)_{deuterium}}{\left(d\sigma(\nu p \to \mu^- \Delta^{++})/dQ^2\right)_{free\ target}}$$

Chi-2 method

$$p \equiv \frac{\sigma^{ANL}_{tot-th}}{\sigma^{ANL}_{tot-exp}} \frac{N^{exp}_{ANL}}{N^{th}_{ANL}}$$

Total # sections

$$\chi^2_{ANL} = \sum_{i=1}^{n_{ANL}} \left(\frac{\sigma^{ANL}_{th}(Q^2_i) - p\sigma^{ANL}_{ex}(Q^2_i)}{p\Delta\sigma^{ANL}_i} \right)^2 + \left(\frac{p-1}{r_{ANL}} \right)^2$$

normalization

Flux uncertainty

$$\sigma^{ANL}_{tot-ex} = \sum_{i=1}^{n_{ANL}} \Delta Q^2_i \sigma^{ANL}_{ex}(Q^2_i) = 0.31278 \times 10^{-38} \mathrm{cm}^2,$$

Analogically the σ_{th} cross section is obtained

$$\sigma^{ANL}_{th}(Q^2_i) = \frac{1}{\Psi_{ANL}} \cdot \frac{1}{\Delta Q^2_i} \int_{Q^2_i - \Delta Q^2_i/2}^{Q^2_i + \Delta Q^2_i/2} dQ^2 \int_{E_{min}}^{E_{max}} dE \Phi_{ANL}(E) \sigma_{th}(Q^2, E), \tag{38}$$

where the $\Phi_{ANL}(E)$ is the neutrino flux, which is normalized to the area restricted by $E_{min} = 0.5$ GeV and $E_{max} = 6.0$ GeV.

$$\Psi_{ANL} = \int_{E_{min}}^{E_{max}} dE \Phi_{ANL}(E) \tag{39}$$

$$\chi^2 = \chi^2_{ANL} + \chi^2_{BNL}$$

$$n_{ANL} + n_{BNL} + 1 - n_{par} - 2$$

$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$$

	M_A [GeV^2]	$C_5^A(0)$	p_{ANL}	p_{BNL}	χ^2/NDF	GoF
free target	0.96 ± 0.04	1.15(fixed)	1.16 ± 0.06	0.98 ± 0.03	0.89	0.62
	0.95 ± 0.04	1.14 ± 0.08	1.15 ± 0.10	0.98 ± 0.03	0.92	0.57
deuteron	0.94 ± 0.04	1.15(fixed)	1.04 ± 0.06	0.97 ± 0.03	0.87	0.66
	0.94 ± 0.03	1.19 ± 0.09	1.08 ± 0.10	0.98 ± 0.03	0.88	0.64

$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{aQ^2}{b+Q^2}\right)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$$

	M_A [GeV^2]	$C_5^A(0)$	p_{ANL}	p_{BNL}	χ^2/NDF	GoF
free target	1.32 ± 0.07	1.15(fixed)	1.24 ± 0.06	0.98 ± 0.03	0.94	0.55
	1.32 ± 0.07	1.09 ± 0.08	1.18 ± 0.10	0.98 ± 0.03	0.96	0.51
deuteron	1.29 ± 0.07	1.15(fixed)	1.12 ± 0.06	0.98 ± 0.03	0.86	0.67
	1.29 ± 0.07	1.13 ± 0.08	1.10 ± 0.10	0.98 ± 0.03	0.89	0.62

For the BNL
Q2 > 0.1!

a=-1.21, b= 2 GeV2

PRELIMINARY

$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$$

	M_A [GeV^2]	$C_5^A(0)$	p_{ANL}	p_{BNL}	χ^2/NDF	GoF
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	0.94 ± 0.03	1.19 ± 0.09	1.08 ± 0.10	0.98 ± 0.03	0.88	0.64

About 11% of difference between ANL and BNL
9-10% of deuteron structure effects

$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{aQ^2}{b+Q^2}\right)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$$

	M_A [GeV^2]	$C_5^A(0)$	p_{ANL}	p_{BNL}	χ^2/NDF	GoF
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	1.29 ± 0.07	1.13 ± 0.08	1.10 ± 0.10	0.98 ± 0.03	0.89	0.62

- ANL
  - Deuteron structure effects seems to more important
- BNL
  - Cut in Q2
  - Higher E beam energy

For the BNL Q2 > 0.1!

a=-1.21, b= 2 GeV2

$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$$

	M_A [GeV^2]	$C_5^A(0)$	p_{ANL}	p_{BNL}	χ^2/NDF	GoF
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	0.95 ± 0.04	1.14 ± 0.08	1.15 ± 0.10	0.98 ± 0.03	0.92	0.57
deuteron	0.94 ± 0.04	1.15(fixed)	1.04 ± 0.06	0.97 ± 0.03	[illegible]	[illegible]
	0.94 ± 0.03	1.19 ± 0.09	1.08 ± 0.10	0.98 ± 0.03	0.88	0.64

Dipole axial mass 0.94 GeV

$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{aQ^2}{b+Q^2}\right)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$$

	M_A [GeV^2]	$C_5^A(0)$	p_{ANL}	p_{BNL}	χ^2/NDF	GoF
free t[illegible]					[illegible]	0.55
	1.32 ± 0.07	1.09 ± 0.08	1.18 ± 0.10	0.98 ± 0.03	0.96	0.51
deuteron	1.29 ± 0.07	1.15(fixed)	1.12 ± 0.06	0.98 ± 0.03	0.86	0.67
	1.29 ± 0.07	1.13 ± 0.08	1.10 ± 0.10	0.98 ± 0.03	0.89	0.62

Radecky (1982) et al.: $M_A = 0.98^{+0.06}_{-0.03}$ GeV

Kitagaki (1990) et al.: $M_A = 1.28^{+0.08}_{-0.10}$ GeV

Axial mass Adler par. 1.29 GeV

For the BNL Q2 > 0.1!

a=-1.21, b= 2 GeV2

ANL

$M_A = 1.00 \pm 0.05$ GeV D = false, Nor = fixed

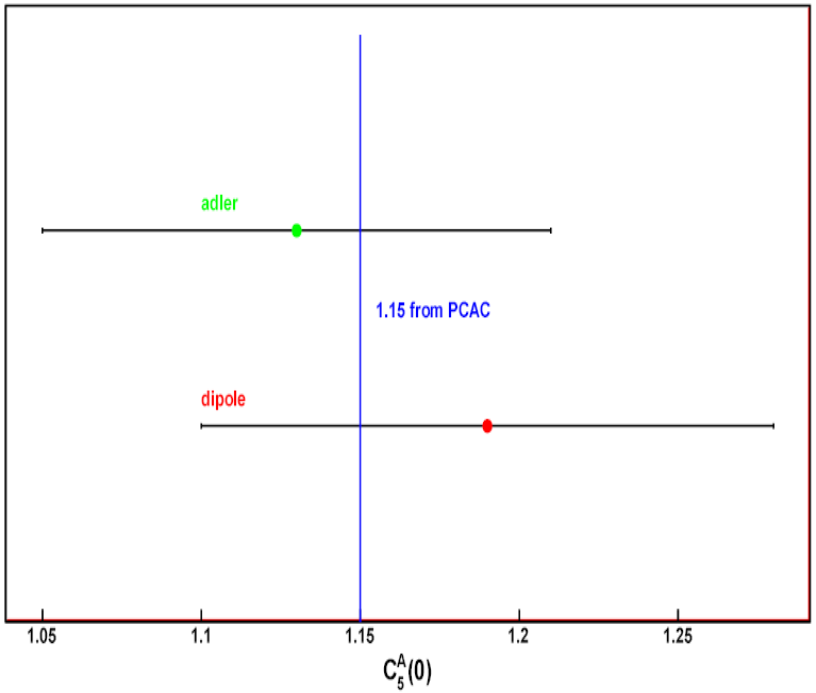
$M_A = 1.11 \pm 0.01$ GeV D = *true*, Nor = fixed

$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$$

	M_A [GeV^2]	$C_5^A(0)$	p_{ANL}	p_{BNL}	χ^2/NDF	GoF
free target	0.96 ± 0.04	1.15(fixed)	1.16 ± 0.06	0.98 ± 0.03	0.89	0.62
	0.95 ± 0.04	1.14 ± 0.08	1.15 ± 0.10	0.98 ± 0.03	0.92	0.57
deuteron	0.94 ± 0.04	1.15(fixed)	1.04 ± 0.06	0.97 ± 0.03	0.87	0.66
	0.94 ± 0.03	1.19 ± 0.09	1.			

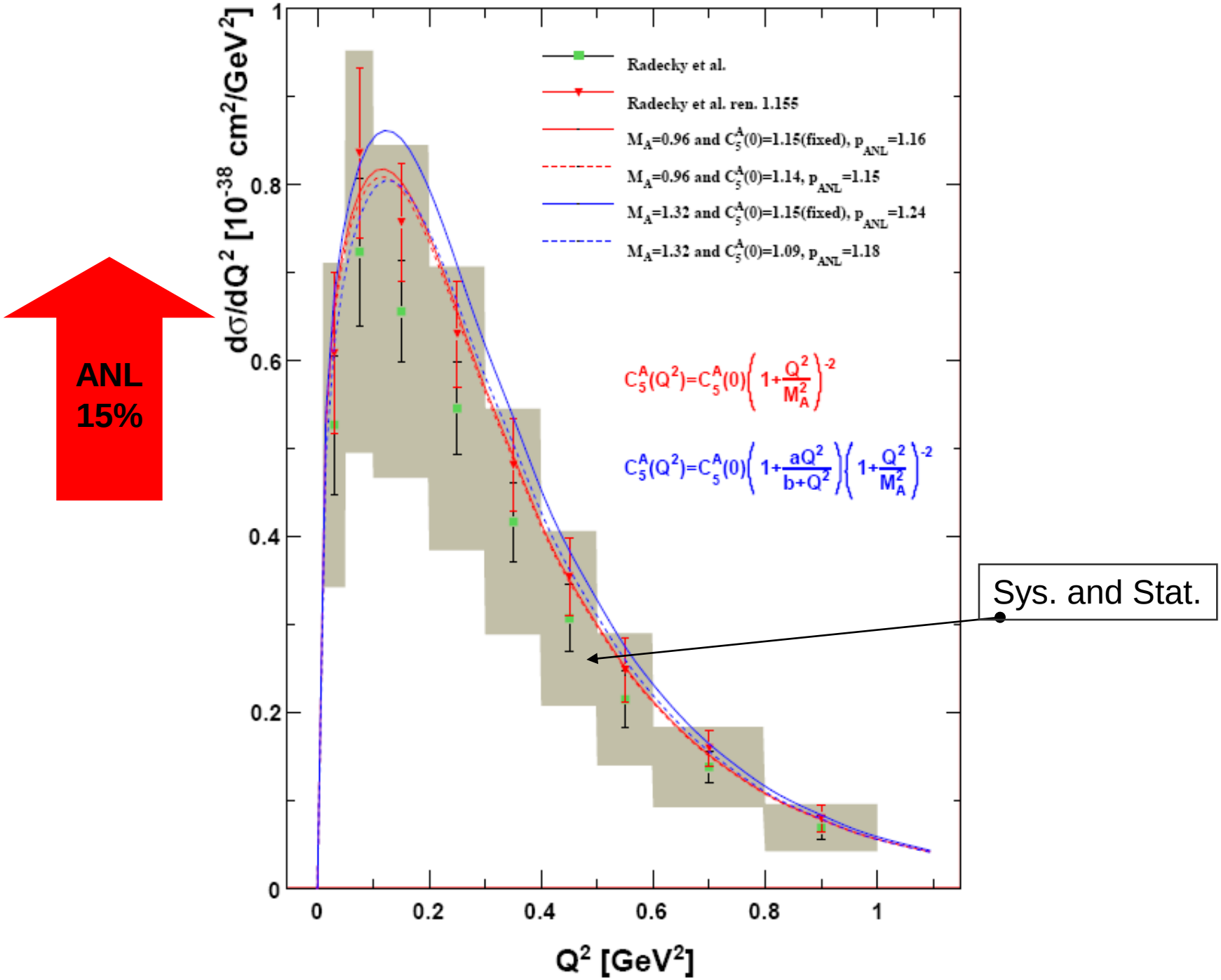
$$C_5^A(Q^2) = C_5^A(0)\left(1+\frac{aQ^2}{b+Q^2}\right)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$$

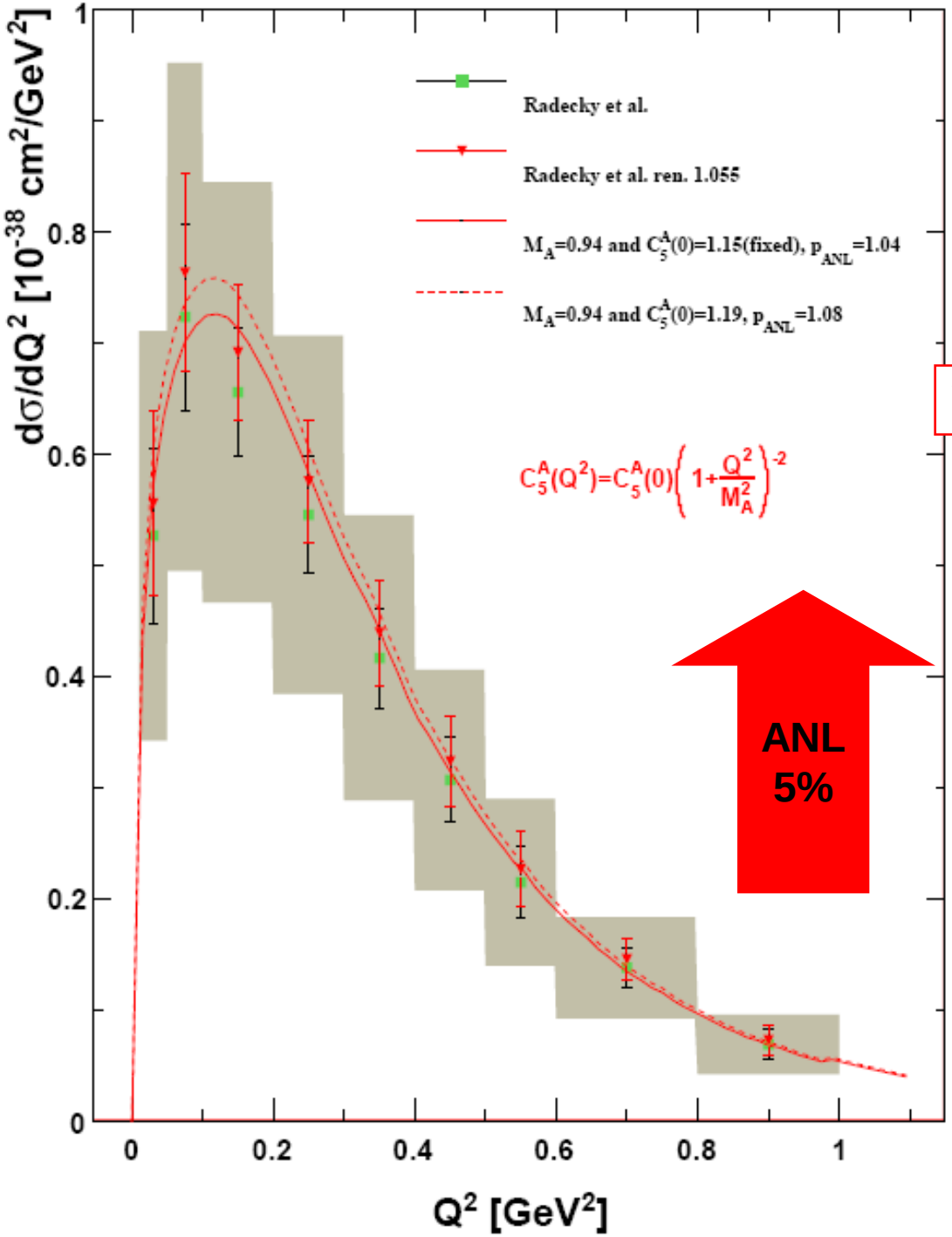
	M_A [GeV^2]	$C_5^A(0)$				
free target	1.32 ± 0.07	1.15(fixed)	1			
	1.32 ± 0.07	1.09 ± 0.08	1			
deuteron	1.29 ± 0.07	1.15(fixed)	1			
	1.29 ± 0.07	1.13 ± 0.08	1.10 ± 0.10	0.98 ± 0.03	0.89	0.62



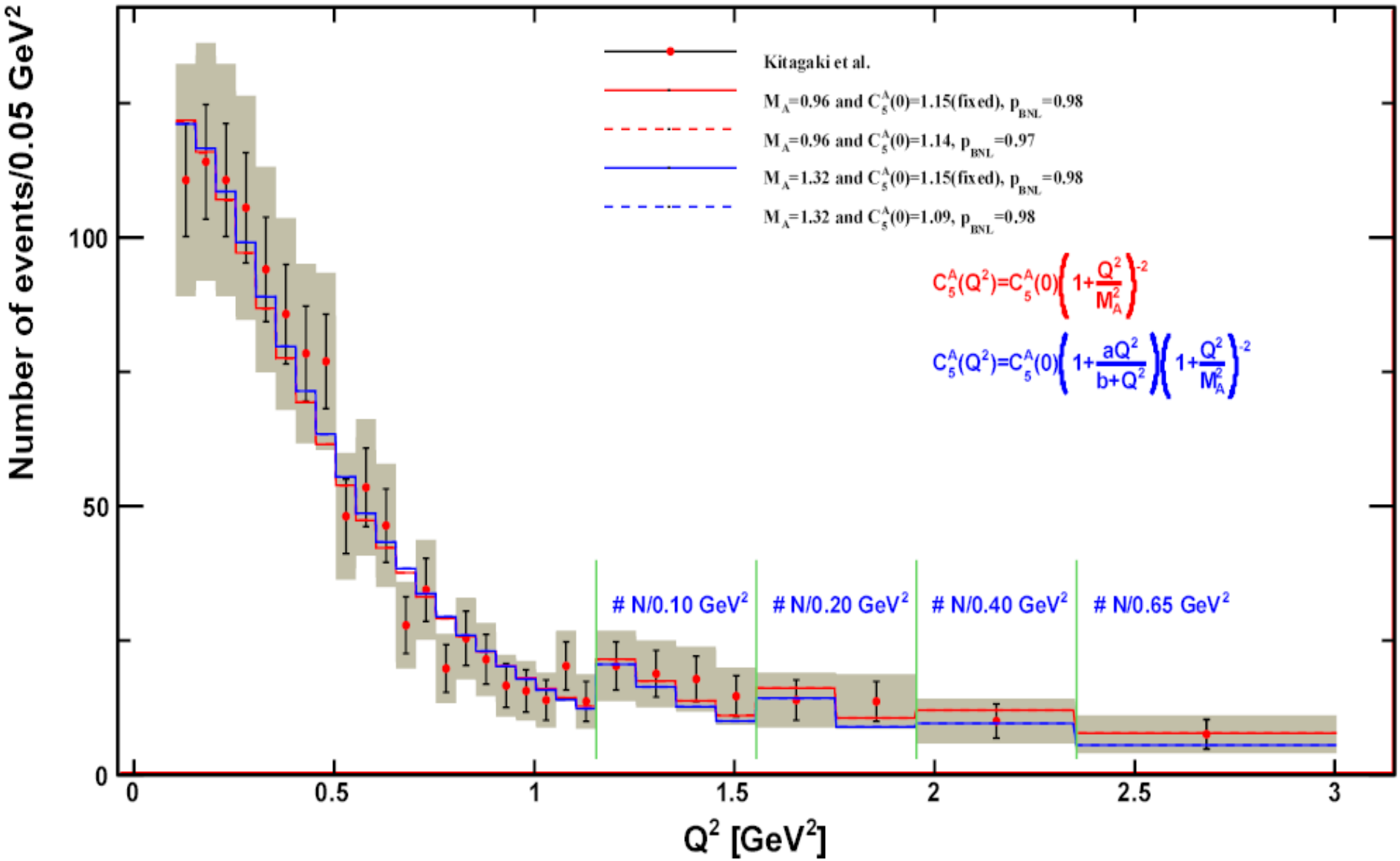
For the BNL
Q2 > 0.1!

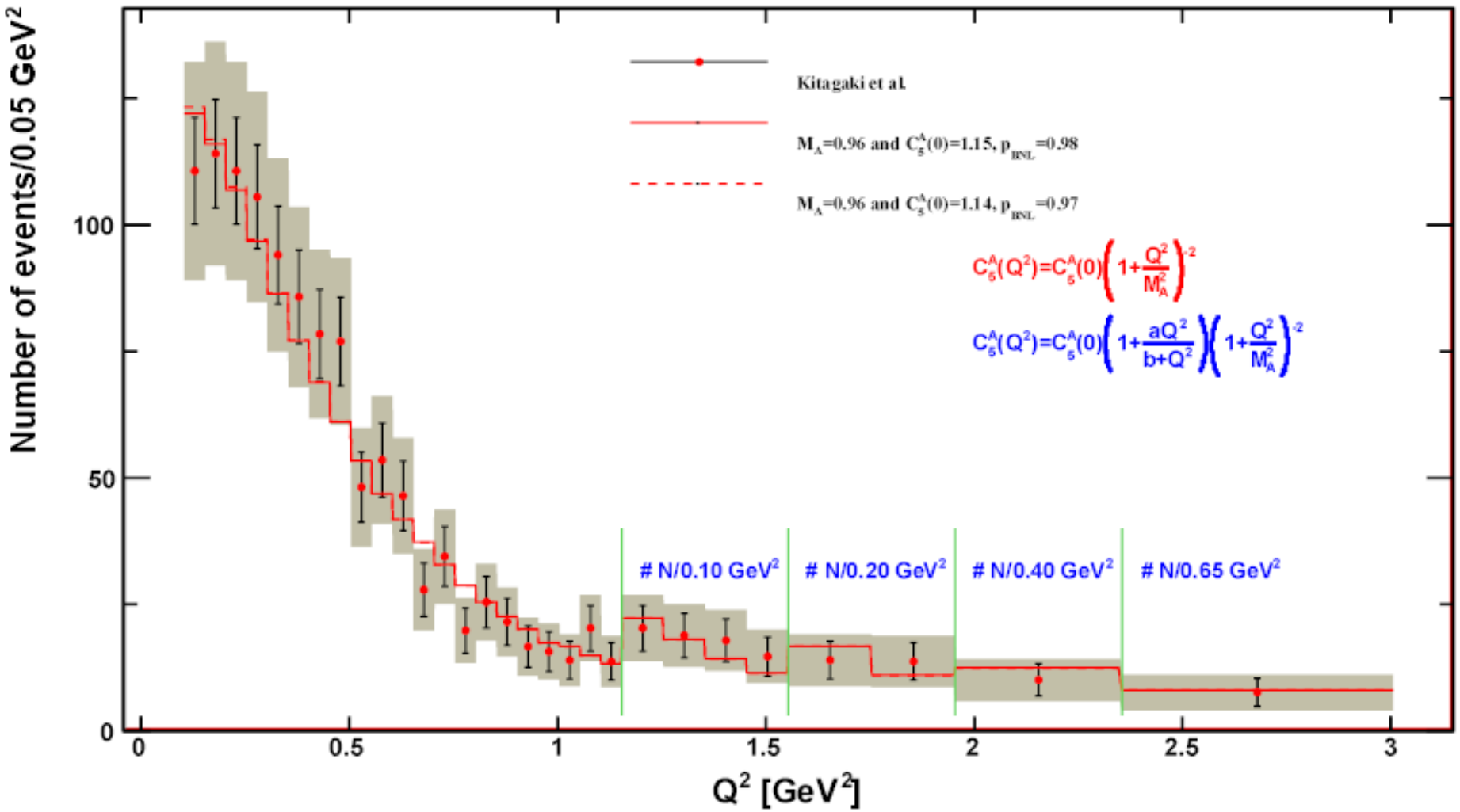
a=-1.21, b= 2 GeV2


ANL
15%
Radecky et al.
Radecky et al. ren. 1.155
M_A=0.96 and $C_5^A(0)$=1.15(fixed), p_{ANL}=1.16
M_A=0.96 and $C_5^A(0)$=1.14, p_{ANL}=1.15
M_A=1.32 and $C_5^A(0)$=1.15(fixed), p_{ANL}=1.24
M_A=1.32 and $C_5^A(0)$=1.09, p_{ANL}=1.18
$C_5^A(Q^2)=C_5^A(0)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$
$C_5^A(Q^2)=C_5^A(0)\left(1+\frac{aQ^2}{b+Q^2}\right)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$
$d\sigma/dQ^2$ [10^{-38} cm^2/GeV^2]
Q^2 [GeV^2]
0
0.2
0.4
0.6
0.8
1
Sys. and Stat.



With deuteron structure effects


Number of events/0.05 GeV^2
Q^2 [GeV^2]
Kitagaki et al.
M_A=0.96 and $C_5^A(0)$=1.15(fixed), p_{BNL}=0.98
M_A=0.96 and $C_5^A(0)$=1.14, p_{BNL}=0.97
M_A=1.32 and $C_5^A(0)$=1.15(fixed), p_{BNL}=0.98
M_A=1.32 and $C_5^A(0)$=1.09, p_{BNL}=0.98
$C_5^A(Q^2)=C_5^A(0)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$
$C_5^A(Q^2)=C_5^A(0)\left(1+\frac{aQ^2}{b+Q^2}\right)\left(1+\frac{Q^2}{M_A^2}\right)^{-2}$
N/0.10 GeV^2
N/0.20 GeV^2
N/0.40 GeV^2
N/0.65 GeV^2
0
50
100
0
0.5
1
1.5
2
2.5
3



With deuteron structure effects

Summary

- Fits are self consistent
- The deuteron structure effects must be accounted especially for ANL data
- The relative difference between ANL and BNL data is around 11 % in the normalization
- The C5A(0) value is consistent with PCAC constraint
- The analysis of uncertainties of cross sections (due to data) for 1π0 production
- A model independent analysis of the data?.....→ Neural Networks?