Results for different current operators (exercise). Show that for free positive energy spinors they are equivalent

$$\Gamma_1^\mu(\boldsymbol{p}_f, \boldsymbol{p}_i) = \gamma^\mu G_M(Q^2) - \frac{P^\mu}{2m} F_2(Q^2)$$

$$\Gamma_2^\mu(\boldsymbol{p}_f, \boldsymbol{p}_i) = \gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2m} F_2(Q^2)$$

$$\Gamma_3^\mu(\boldsymbol{p}_f, \boldsymbol{p}_i) = \frac{P^\mu}{2m} F_1(Q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2m} G_M(Q^2)$$

On shell nucleons

- We can put artificially ‘on shell’ the initial and final nucleon by forcing their wave functions to fullfill ‘half’ the *free* Dirac equation
- Gordon transformation will be valid for these EMA spinors, and matrix element should factorize into the ones for free nucleons
- Show that the previous current operator are equivalent for EMA (on-shell) positive energy spinors. Show that they verify free Dirac equation

- $$\psi_{RDWIA}(\vec{p}) = \begin{pmatrix} \psi_{up}(\vec{p}) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M+S-V}\psi_{up}(\vec{p}) \end{pmatrix} \rightarrow$$

$$\psi_{EMA}(\vec{p}) = \begin{pmatrix} \psi_{up}(\vec{p}) \\ \frac{\vec{\sigma}\cdot\vec{p}_{as}}{E_{as}+M}\psi_{up}(\vec{p}) \end{pmatrix}$$

Superscaling QE (e,e’) predictions (SUSAEE)

Codes sigsusa.linux, sigsusa.cygwin.exe, sigsusa.win.exe

Input file sigsusa.in:

1 target nucleus (1: 12C, 2: 16O, 3: 40CA, 4: 56FE)
680. -36. ebeam (MeV) and q (if <0, assume it is the scattering angle in degrees)
1. 300. 10. win wfin wstep (energy transfer range)

Output file (standard output)

w(GeV) sigma(nbarn/sr/GeV) psi’ f(psi’) q (MeV/c) theta

References:
Phys.Rev.Lett.95:252502,2005, Phys.Rev. C68 (2003) 048501, Phys.Rev.Lett.100:052502,2008, Phys.Lett.B653:366-372,2007, PRC60,065502

RFG (e,e') predictions

Codes rfgee.f, rfgee.cygwin.exe, rfgee.linux, rfg.win.exe
Input file rfg.in:
(see next slide)

Output file sigma.out:

w(GeV) sigma(nbarn/sr/GeV) psi' f(psi') q (MeV/c) theta

References:

Many references for RFG. This version is my own RFG that is somewhat described in my internal report (inclusiv.pdf notes) and in Nucl.Phys.A602:263-307,1996 and I've employed in Phys. Rev. Lett. 74 (1995) 4993, Phys.Rev.C52:3399-3415,1995

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC Comienzo del caso....
output file
pwdwia.out
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
Zinc, Amass, Zfin, Amass fin.    kappa!!!! (described in Katori Ph.D.)
6.        12.        5.   11.008503    1.000
XXXXXXXXXXXXXXXXXXX     Iff ireac (0 for EM)         not used   neut(1.) or antineut (-1.)
0   0    0     0.0                           1             0                    0                 1.
win, wfin wstep q or thet     Pmfermi, Pauli Blocking Energy (MeV)
0.01 300.    1.        -36           225.5        27.
ekin,     Binding Energy (MeV)   XXXX
680.      27.0                           0.0
XXXXXXXXXXXXXXXXXXXXXX
0
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC Opciones del programa
XXXXXX   N_points cosmin    cosmax           print
0.  0.             100    -0.9999999999   0.9999999999      0
nuc. intermediate files (nuc00,nucll)
emuwork:nuc00.dat
emuwork:nucll.dat
radial integrals interm. files (rad00,ll1,ll2)
emuwork:rad00.dat
emuwork:radll1.dat
emuwork:radll2.dat
output file for pm-cross-section
pwcrs.out
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC Opciones del calculo
XXXXXXX, Icc (1-cc1, 2-cc2 < 0, impose cc)      F2s     G1s
 0    0            -1                                        0   -0.   -0.0
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC Fin del caso....
```

RMF calculations for nucleonic response

RMF-CC0-60-680, RMF-CC0-60-620, RMF-CC0-36-680, RMF-CC0-36-560
RMF-CC0-36-480, RMF-CC0-60-560, RMF-CC0-60-519, RMF-CC0-60-440, RMF-CC0-13.54-1500

Files: protons.crs, neutrons.crs, total.crs and others

Format of the files:

w(MeV) sig1 sig2 sig3 sig_noFSI

USE ONLY sig2 or sig3 or sig_noFSI

Look out UNITS: dsigma/domega (nb/MeV/sr2)

Described in my talk, also Phys.Rev. C68 (2003) 048501, Phys.Rev.Lett.95:252502,2005

Inclusive (e,e’) data

File 12C-data.dat contains most data for 12C in (e,e’)

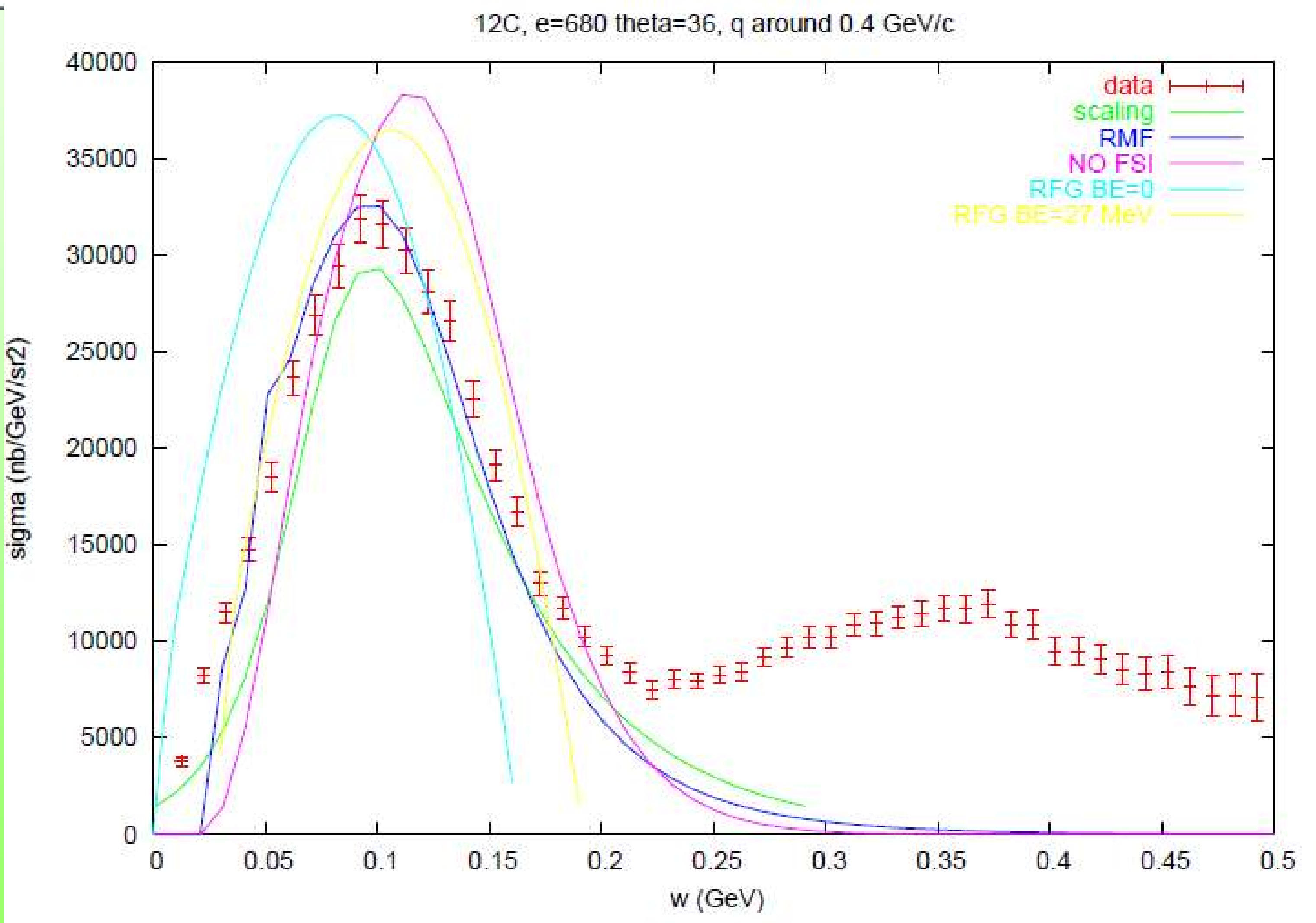
Format of file:

Z A beam-energy (GeV) angle w(GeV) dsigma/domega err ref.

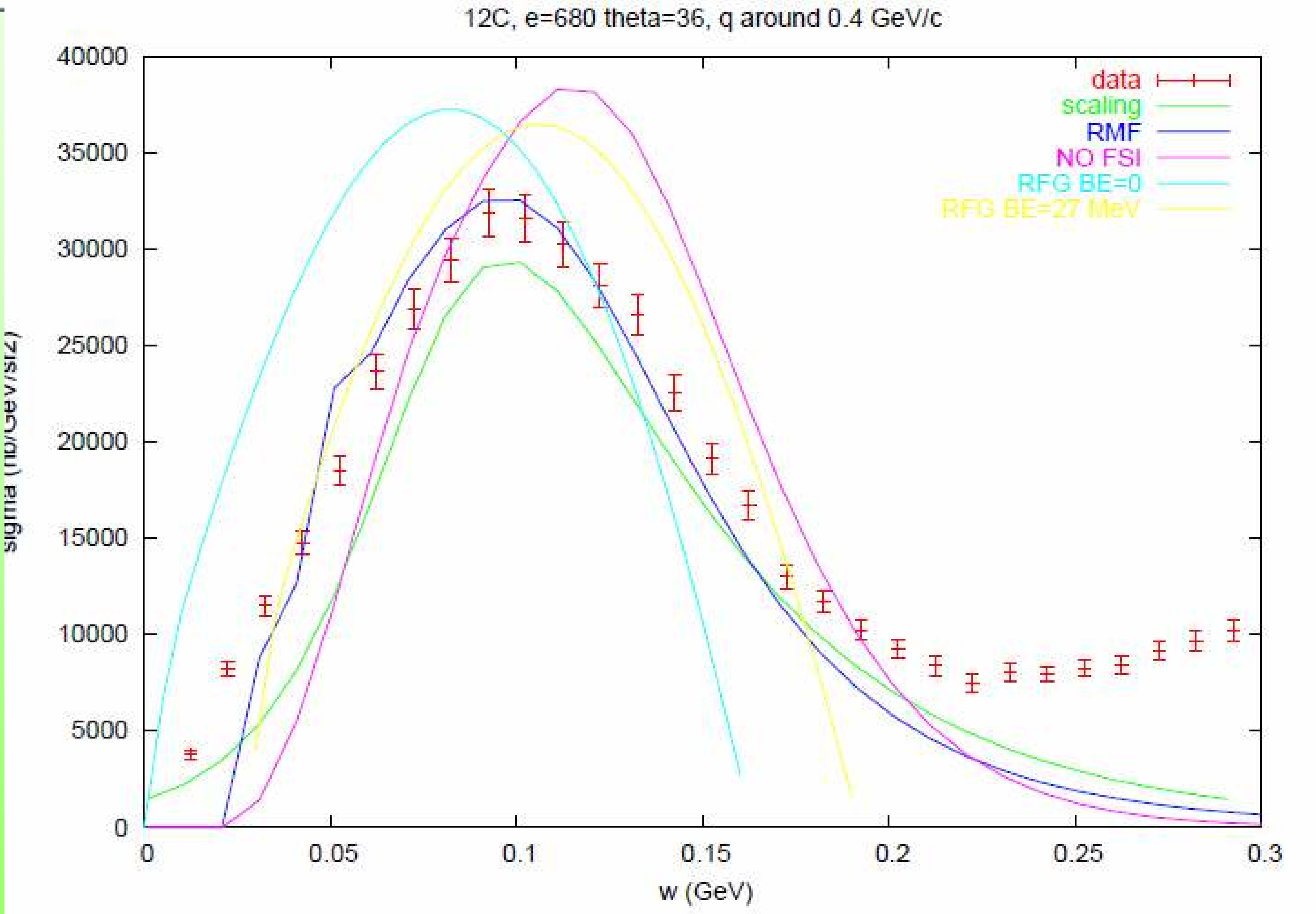
Easy to get from the Virginia Welcome to Quasielastic Electron Nucleus Scattering Archive (thanks to D. Day and many others):

http://faculty.virginia.edu/qes-archive/C12/C12-index.php

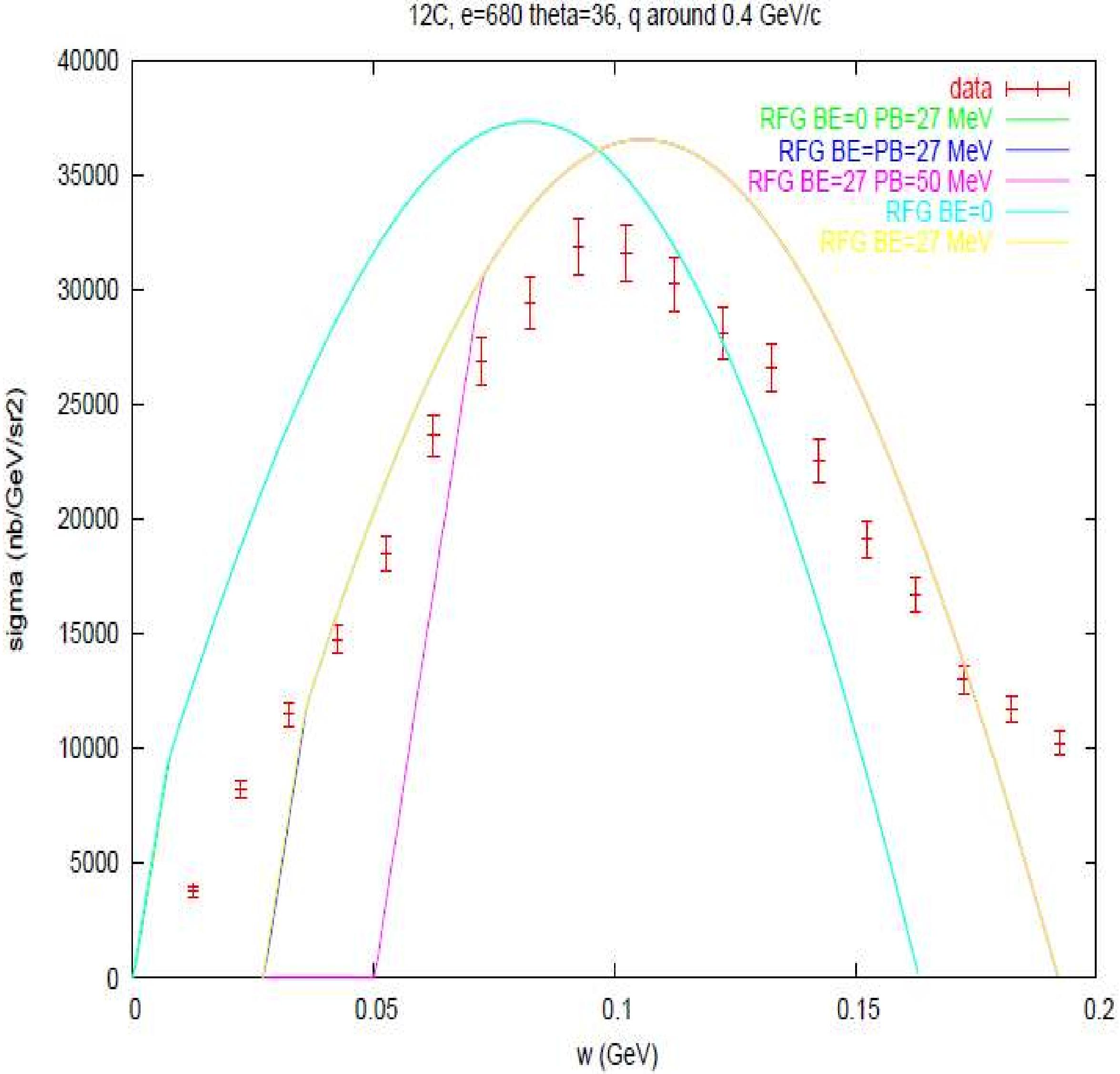
Comparison to experiment



Zoom into the nucleonic peak



Effect of RFG parameters

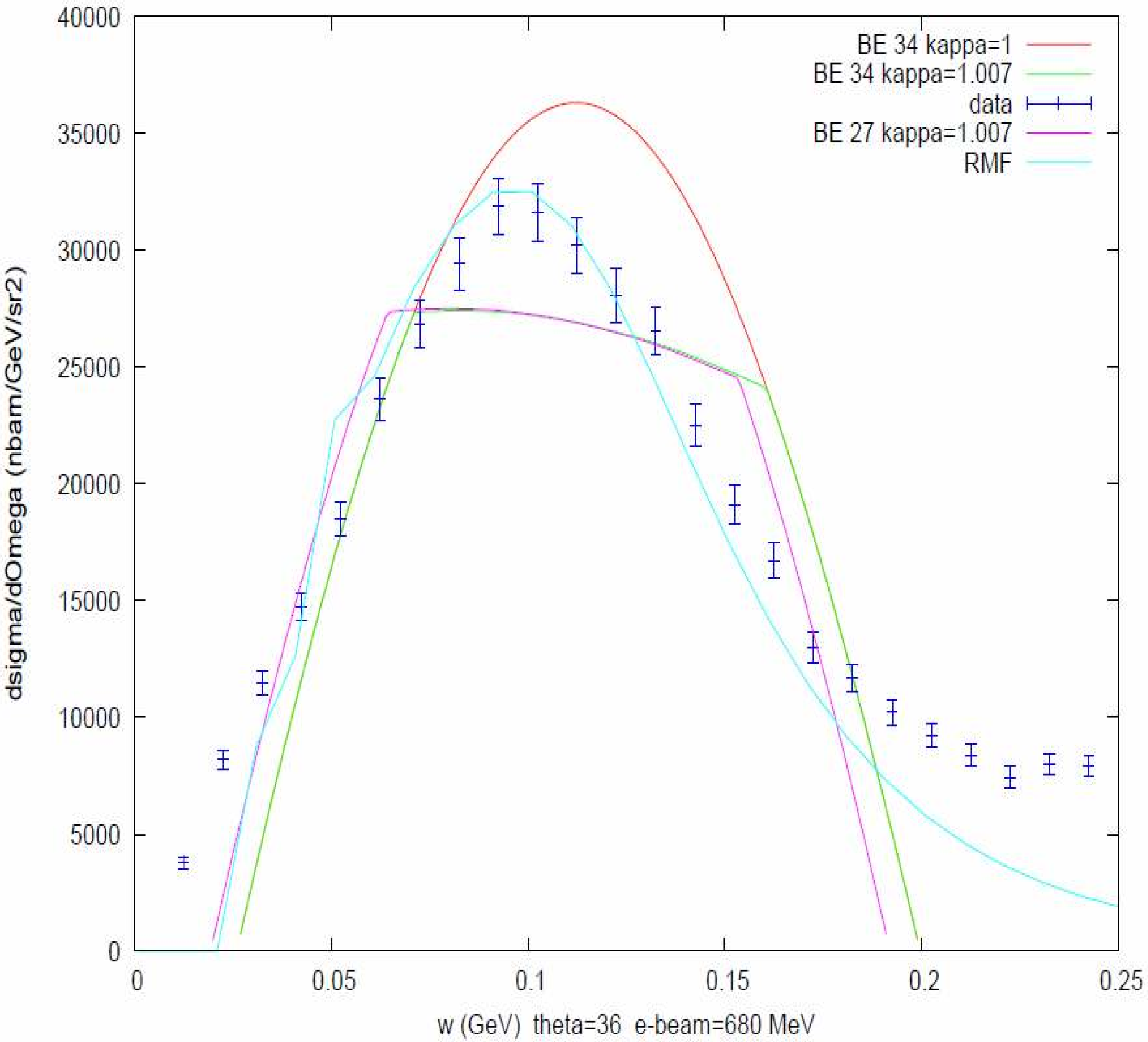


PB is negligible except for very small q or unrealistical values of PB. PB just avoids receiving energy from the nuclei

BE puts the peak in the right position and breaks Gauge invariance and puts the nucleons off-shell

RFG parameters give some handle to tune thenuclear model, but not a very good one!

Effect of RFG parameters

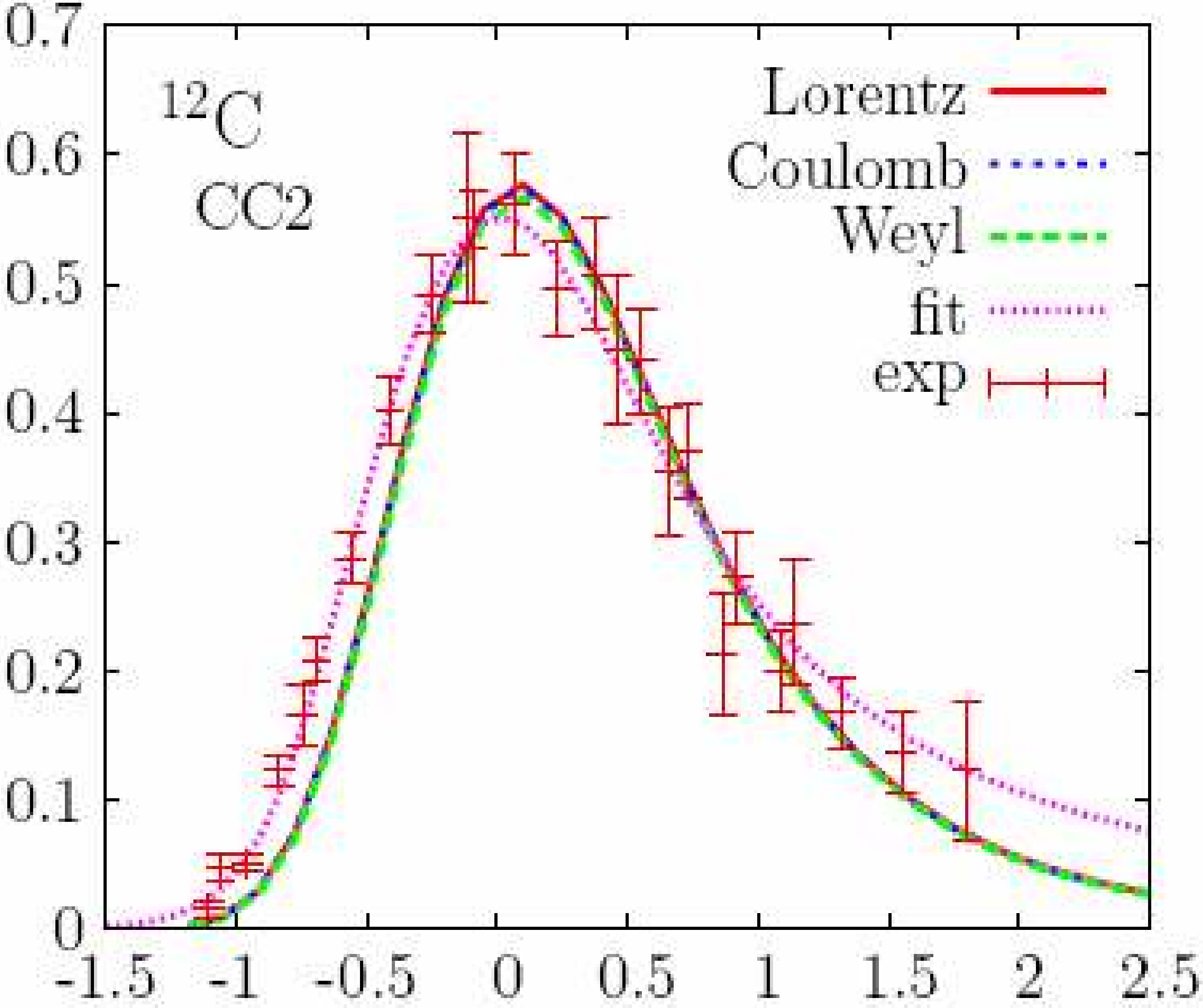


Kappa is a new version of PB that has little to do with actual Pauli blocking concept

Increased BE shifts the peak additionally, which is not worse or better

kappa parameter gives an additional handle to tune the nuclear model!

Current conservation / Continuity Equation / Gauge Invariance



If same effective or whatsoever mean field potential is employed for the initial and final state, then the matrix element is independent on the Gauge choice at the mean field level.

For RMF it yields the same results for *on-shell* and *off-shell* spinors if using cc2 or cc3 current operators

Suplement the notes by adding the charged current case

Complutense

$$\left(d\sigma^{Z^0/W^\pm}\right)_{\mathrm{Free}} = \delta^{(4)}(k_i^\mu - k_f^\mu + P_I^\mu - P_F^\mu)\,\sigma^{Z^0/W^\pm}\,\frac{1}{4\epsilon_f^2 E_I E_F}\,\omega_{\mu\nu}W^{\mu\nu}\,d^3\vec{P}_F d^3\vec{k}_f$$

$$\sigma^{Z^0} = 16\,\epsilon_f^2\cos^2(\theta/2)\left[\frac{g^2}{4\pi}\right]^2$$

$$\sigma^{W^\pm} = 16 k_f^2\left[\frac{g^2}{4\pi}\right]^2$$

The hadronic part does not need to be computed at every point

$$\omega_L W_L = \frac{1}{4\epsilon_i k_f}\Bigg\{\left[(\epsilon_i+\epsilon_f)^2 - |\vec{k}\,|^2 - m_l^2\right]|\rho|^2 + \left[\frac{(\epsilon_i^2-k_f^2)^2}{|\vec{k}\,|^2} - \omega^2 + m_l^2\right]|J_k|^2 - \left[\frac{2(\epsilon_i+e_f)(\epsilon_i^2-k_f^2)}{|\vec{k}\,|} - 2\omega|\vec{k}\,|\right] Re\,(\rho^* J_k)\Bigg\}$$

$$\omega_T W_T = \Bigg\{\cancel{\frac{\epsilon_i k_f \sin^2\theta}{2|\vec{k}|^2}\cos(2\phi_F)\left(|J_\parallel|^2 - |J_\perp|^2\right)} + \left[\frac{\epsilon_i k_f \sin^2\theta}{2|\vec{k}\,|^2} - \frac{1}{2}\left(\frac{-\epsilon_f}{k_f} + \cos\theta\right)\right]\left(|J_\parallel|^2 + |J_\perp|^2\right)\Bigg\}$$

$$\omega_{TT'}W_{TT'} = -\frac{1}{|\vec{k}\,|}\left(\frac{\epsilon_i\epsilon_f}{k_f} + k_f - (\epsilon_i+\epsilon_f)\cos\theta\right) Im\,(J_\parallel J_\perp^*)$$

L, T and TT' are the only responses that contribute if no nucleon is observed

RFG (ν_μ,μ^-) predictions

Codes muonx.f, muonx.cygwin.exe, muonx.linux, muonx.win.exe

Input file muonx.in:

(see next slide)

Output file musigx.out:

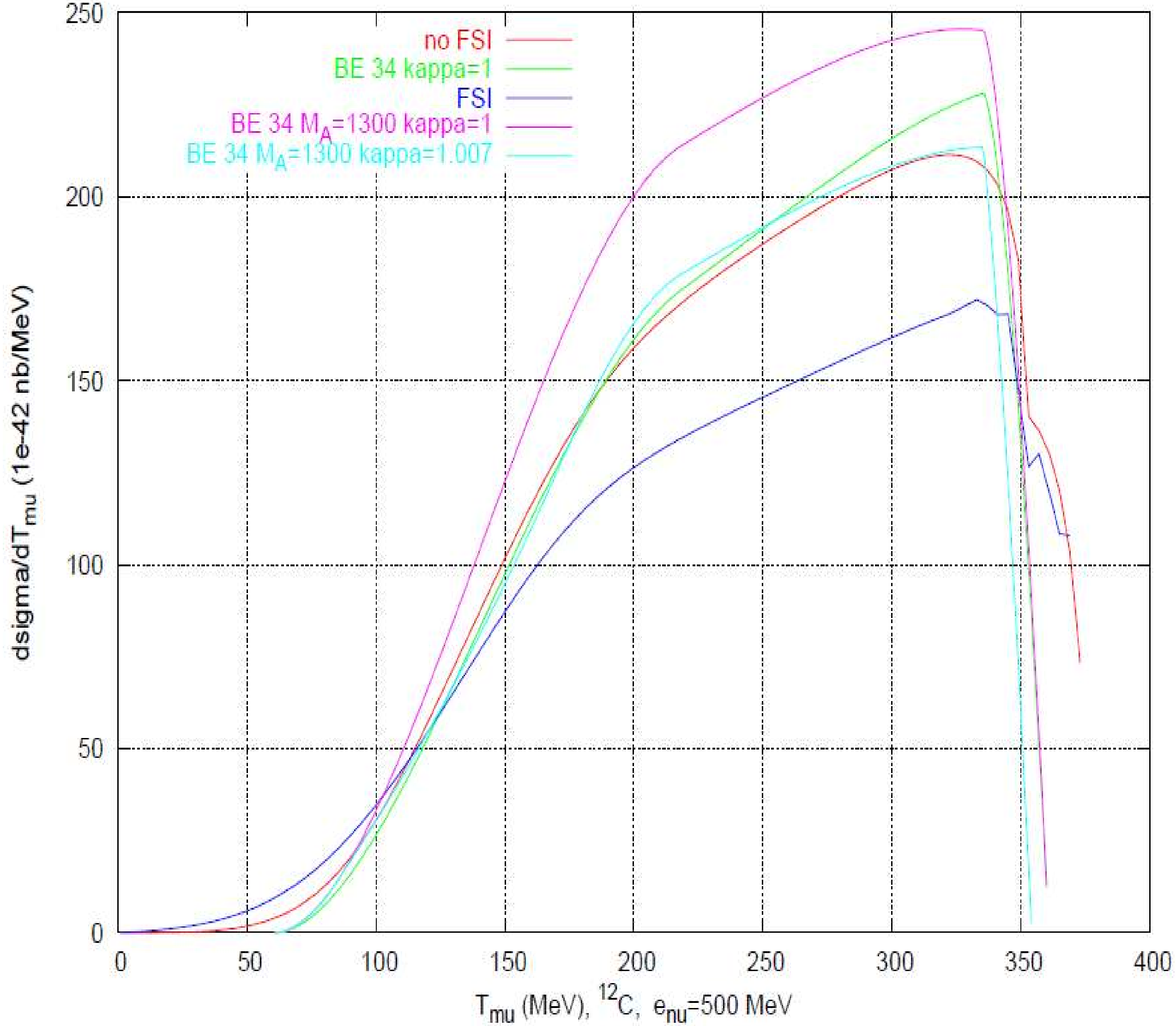
T_mu (MeV) e_nu(MeV) sigma(10^{-42} cm^2 /MeV)

References:

Many references for RFG. This version is my own RFG that is somewhat described in my internal report (inclusiv.pdf notes) and in Nucl.Phys.A602:263-307,1996 and I've employed in Phys. Rev. Lett. 74 (1995) 4993, Phys.Rev.C52:3399-3415,1995

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC Comienzo del caso....
output file
mec.out
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
Zinc, Amass, Zfin, Amass fin. axial_mass (MeV)  kappa
6.        12.     5.   15.008503      1020                  1.007
xxxxxxxxxxxxxx  Iff ireac icharg (1 p, 2 n)neut(1.) or antineut (-1.)
1   2    3     100.0     1     1     1                   1.
win,   wfin wstep  PAULI ex.     Pmfermi, mass muon   (MeV) (105.66) vcoul
0.01   1500.0  1.    -1.0                  220.    105.066            -0.0
ekin,   Bind. En. (MeV)   tick
500.    34.0            0.05
Bound state file
c12hs.1pn
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC Opciones del programa
xxxxxxxxxxxxx N   cosmin        cosmax         imp   nmec(if 0 no mec)
200.    12.         100   -0.99999  0.99999         0         0     0   1
nuc. intermediate files (nuc00,nucll)
emuwork:nuc00.dat
emuwork:nucll.dat
radial integrals interm. files (rad00,ll1,ll2)
emuwork:rad00.dat
emuwork:radll1.dat
emuwork:radll2.dat
output file for pm-cross-section
pwcrs.out
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC Opciones del calculo
IPL, IPT, Icc (1-cc1, 2-cc2 < 0, impose cc) iwoper
 0    3   2                               -1   -0.0  0.
```

Effect of RFG parameters



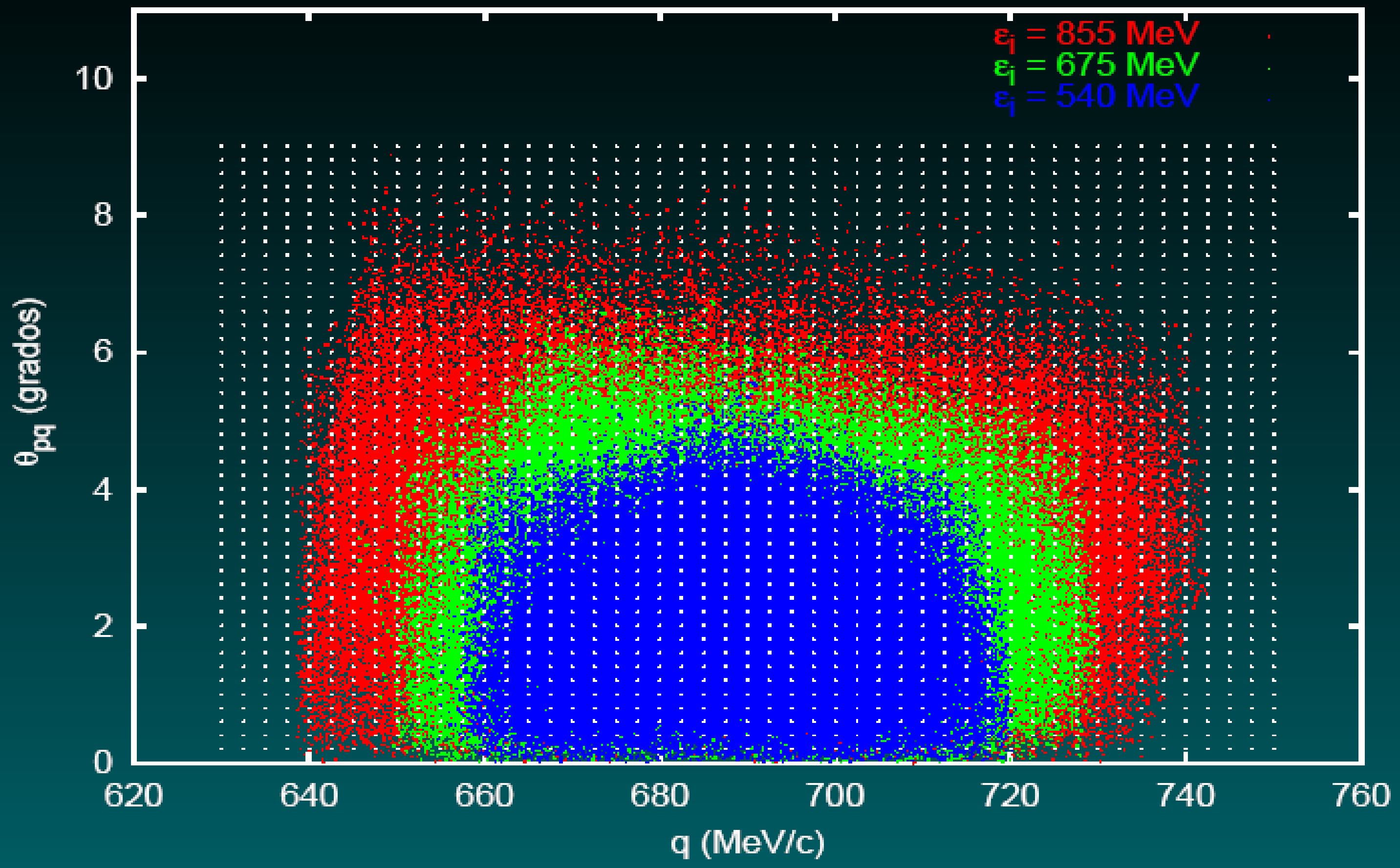
Increasing the axial mass changes the cross-section significantly

kappa parameter returns it to a place near ordinary RFG

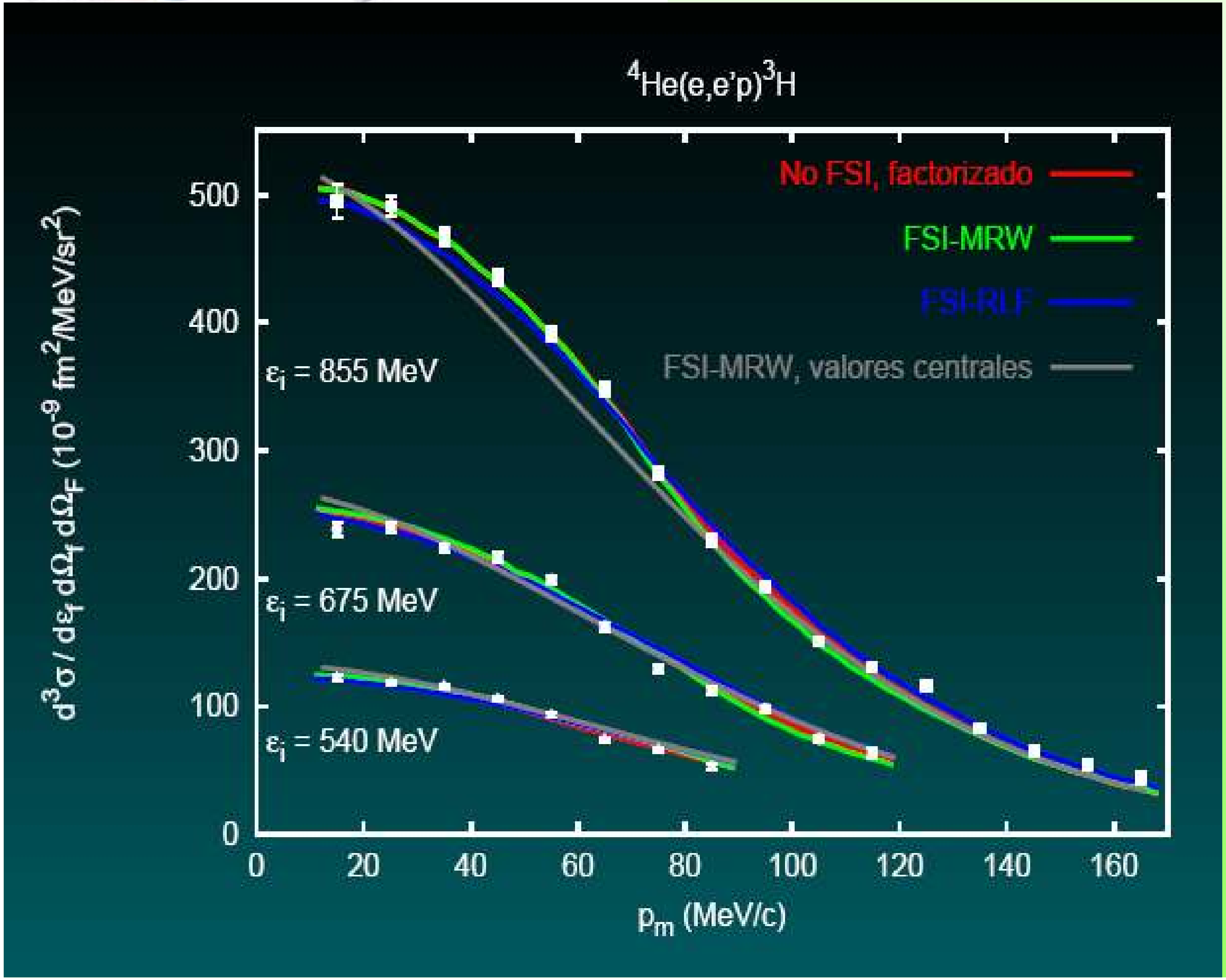
There is still the effect of FSI

Other handles to tune: H_p

Event map

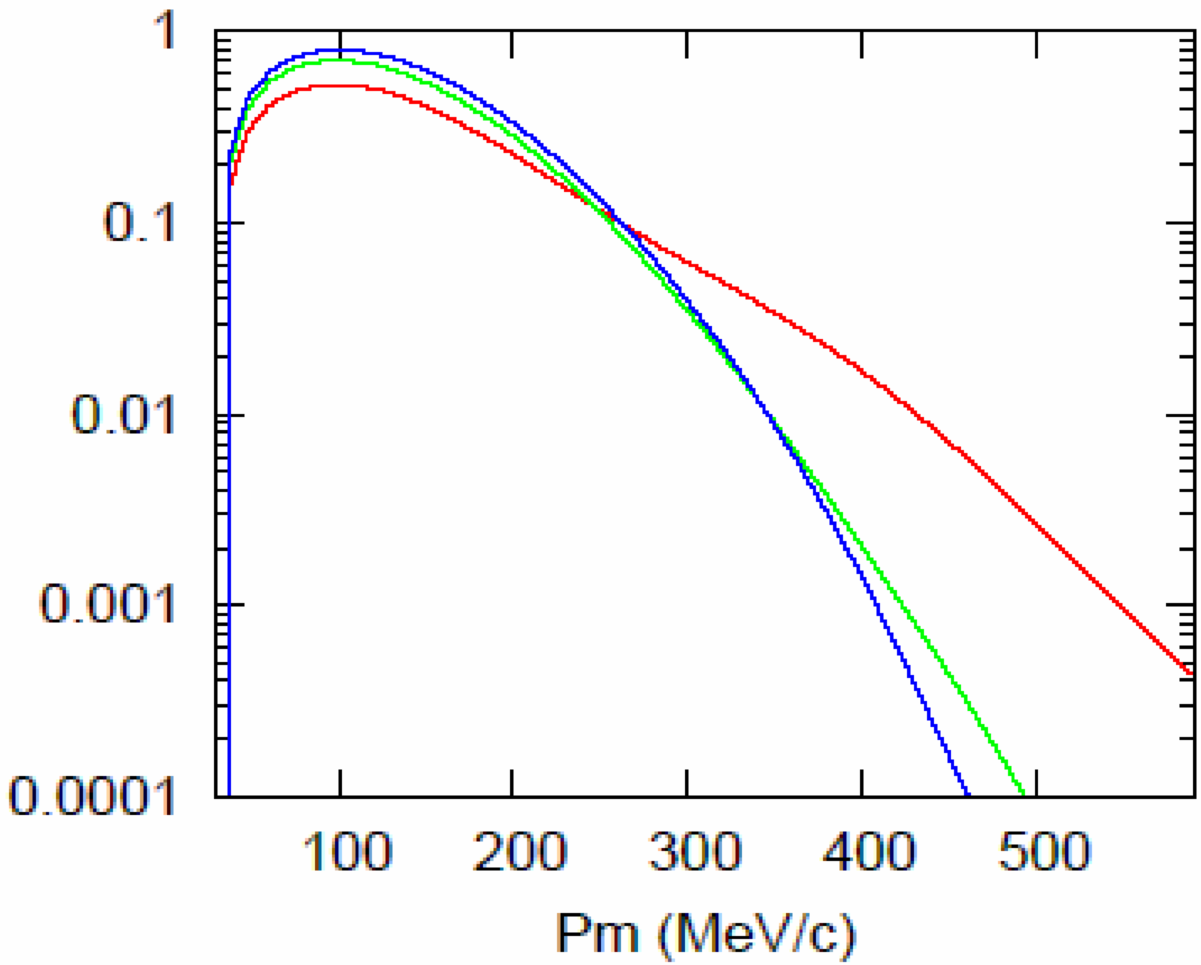


ε_i = 855 MeV
ε_i = 675 MeV
ε_i = 540 MeV
θ_{pq} (grados)
q (MeV/c)
0
2
4
6
8
10
620
640
660
680
700
720
740
760



Data from Mainz A1, Florizone Ph.D. Thesis (1999)

Effects of FSI



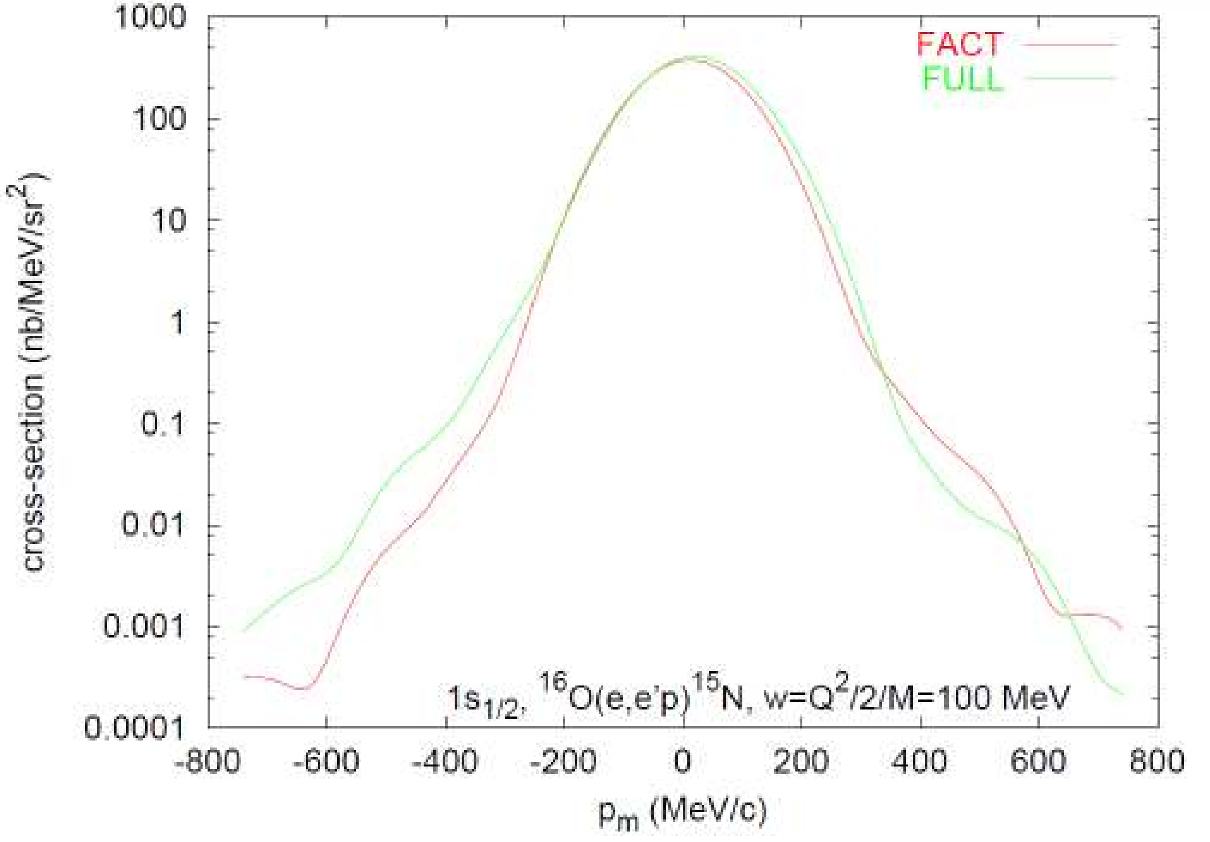
Effective p3/2 momentum distribution taken from a factorized calculation

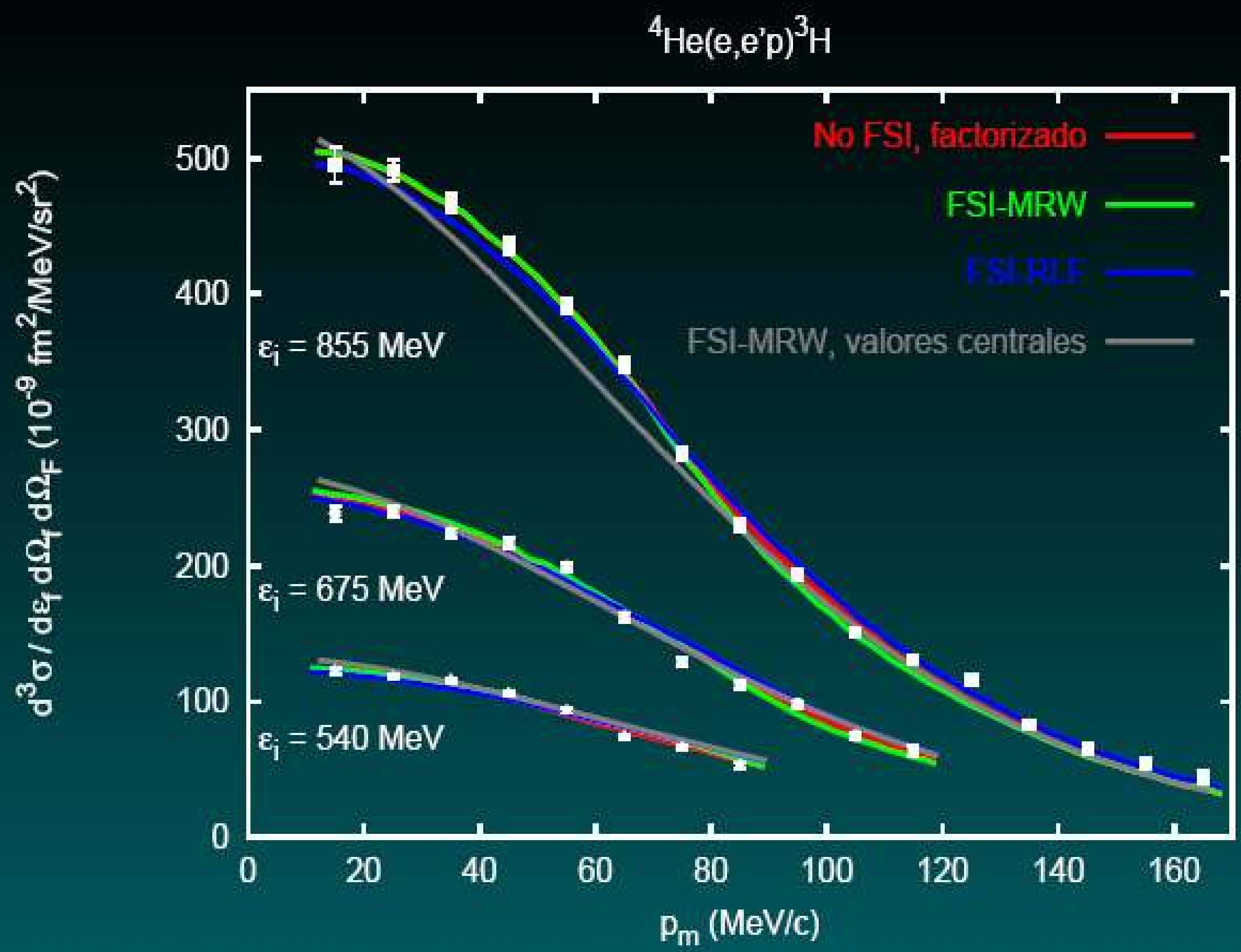
Blue line has no FSI effects green on has FSI effects as given by a nonrelativistic approach

Red one includes FSI given by the RMF

Factorization approach

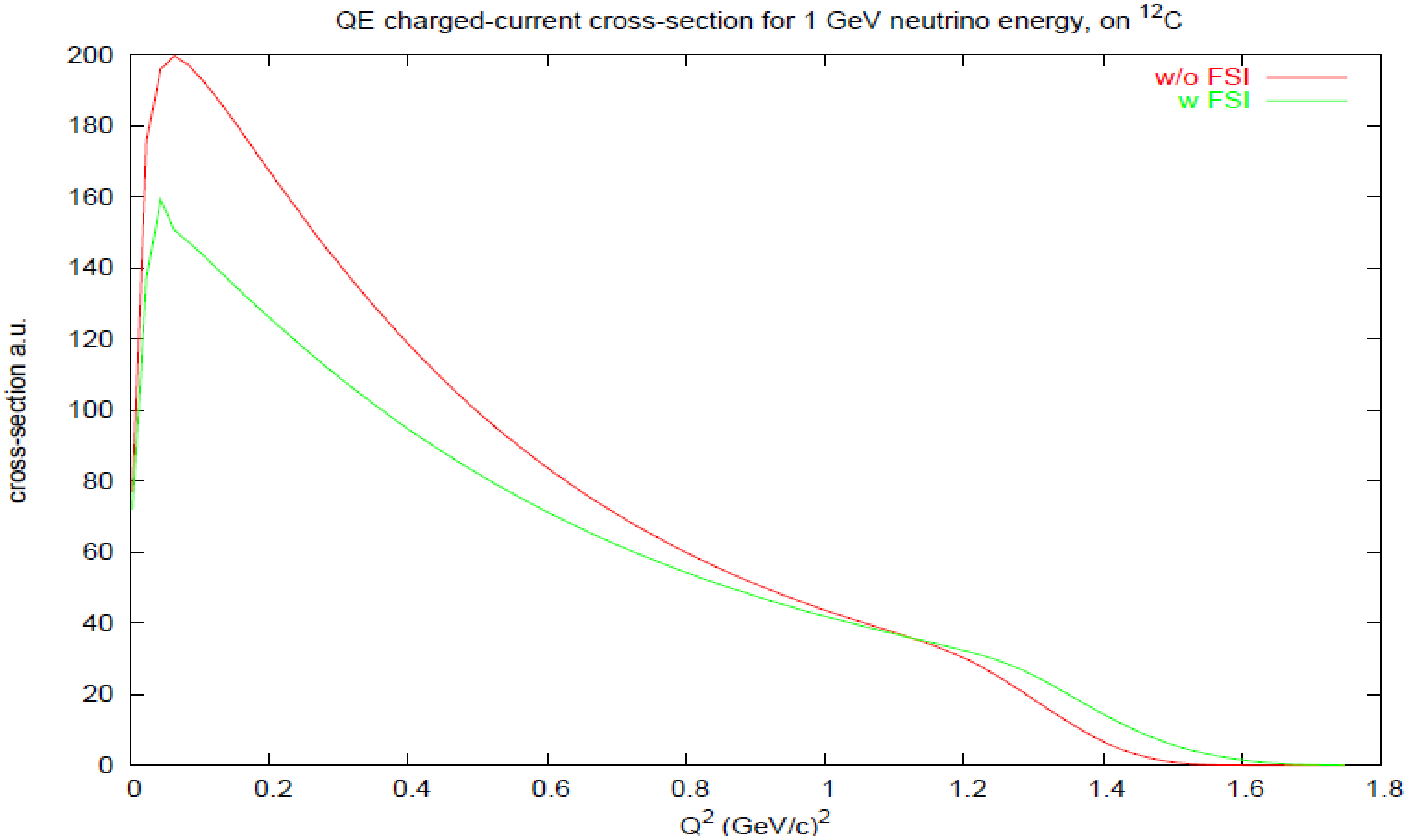
$$\frac{d^5\sigma}{d\Omega_e d\varepsilon' d\Omega_F} = K\sigma_{ep} S(E_m, \vec{p}_m) \qquad \rho^{exp}(\mathbf{p}_m) = \frac{\left(\frac{d\sigma}{d\varepsilon_f d\Omega_f d\Omega_F}\right)^{exp}}{E_F p_F f_{rec}\sigma_{ep}}.$$





Data from Mainz A1, Florizone Ph.D. Thesis (1999)

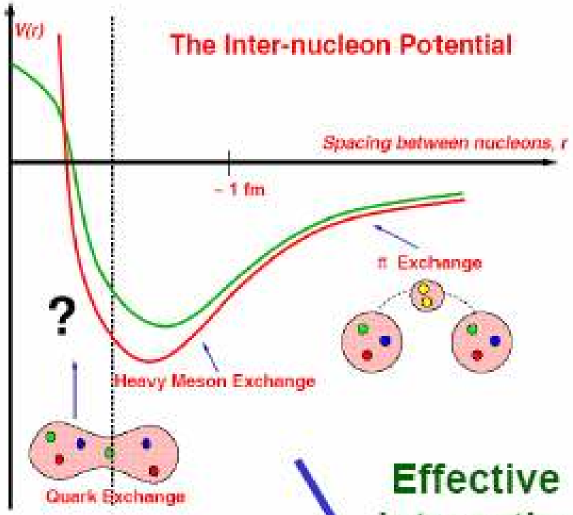
RMF calculations: Nuclear effects have a noticeable impact for 1 GeV neutrino energy. I'll prepare codes for the CC case



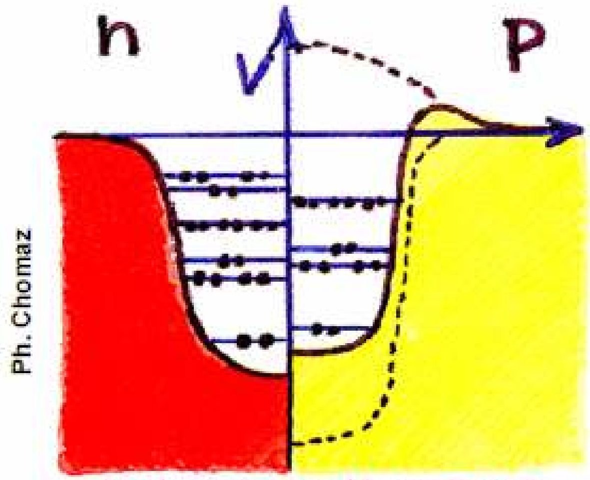
A survey of the relativistic mean field approach

B. D. Serot and J. D. Walecka, The relativistic nuclear many body problem. Adv. Nuc. Phys., 16:1, 1986.

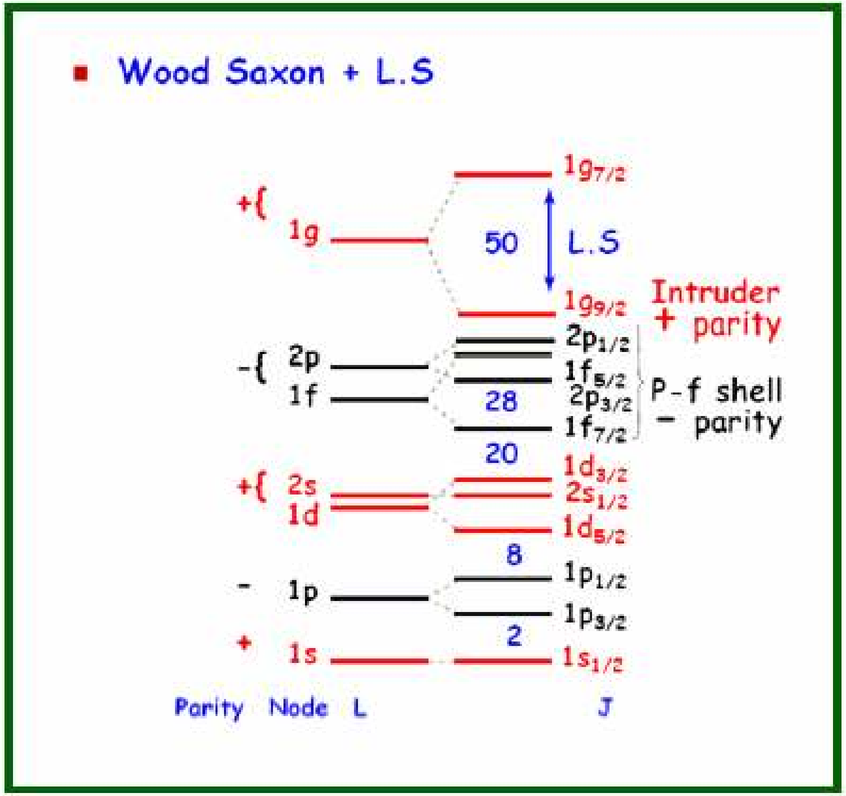
Mean Field Model of Nuclei



Effective interaction



Nucleons in a self-consistent mean-field



- fermion system at low energies
- suppression of collisions by Pauli exclusion
- independent particle motion
- shell structure
- mean field approximation

Non relativistic mean field

•Small potentials (a few tents of MeV), of the order of the binding energy in nuclei

•Separate Central + Spin-Orbit potential

•Importance of Fock (exchange) terms and correlations beyond the mean field

(In non-relativistic models the saturation arises from the interplay between a long range attraction and a short range repulsion, so strong that it **<u>is compulsory to take short range correlations</u>** into account)

Relativistic mean field

- What is the role that relativity plays in nuclear systems?

- The ratio of the Fermi momentum over the nucleon mass is about $k_F/M = 0.25$. Nucleons move with at most about 1/4 of the velocity of light. Only moderate corrections from relativistic kinematics are expected
- Strong potentials (a few hundreds of MeV's). Small binding energy is just the 'tip' of the iceberg
- Spin-Orbit potential implicit in the relativistic formalism

- However, there exists a fundamental difference between relativistic and non-relativistic dynamics: a genuine feature of relativistic nuclear dynamics is the appearance of large scalar and vector mean fields, each of a magnitude of several hundreds MeV. The scalar field S is attractive and the **vector field V is repulsive**

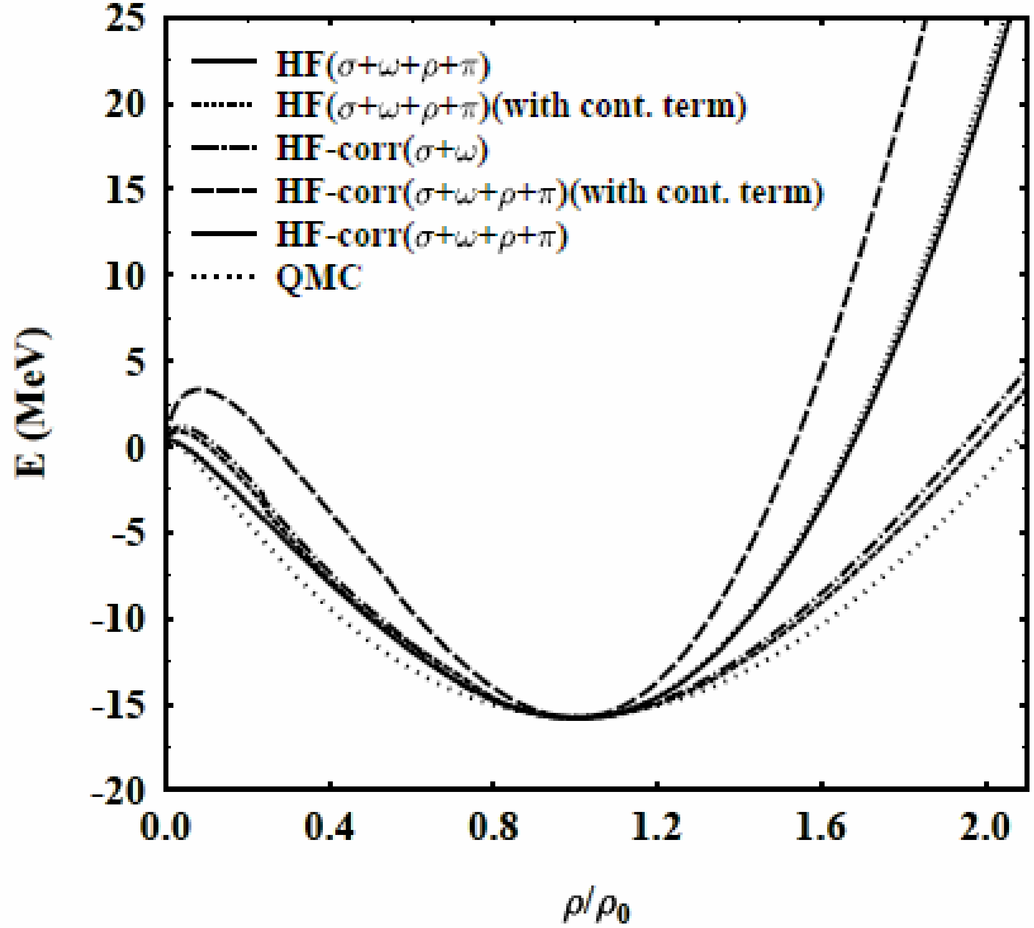
- In relativistic mean field (RMF) theory, both sign and size of the fields are enforced by the nuclear saturation mechanism

•In relativistic mean field models, the parameters are phenomenologically fitted to the saturation properties of nuclear matter

•Short range correlations are not needed to get the right saturation properties

• In this approach short range correlation effects may be accounted for, to some extent, by the model already at mean field level

•Formally, the scalar and vector potentials are usually implemented via (scalar and vector) meson exchanges



arXiv:nucl-th/0602059 v1 21 Feb 2006

P.K. Panda, Joao da Providencia and Constança Providencia

Formally, one can build a field theory. It can seem very convincing...

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left(\gamma_\mu\left(\mathrm{i}\partial^\mu - g_\omega\omega^\mu - g_\rho\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu\right) - \left(M + g_\sigma\varphi\right)\right)\psi \\ &+ \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m_\sigma^2\varphi^2 - \frac{1}{3}g_2\varphi^3 - \frac{1}{4}g_3\varphi^4 \\ &- \frac{1}{4}\omega_{\mu\nu}\cdot\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2 \\ &- \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu\end{aligned}$$

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial_\mu\phi}\right) - \frac{\partial\mathcal{L}}{\partial\phi} = 0.$$

$$\left(\Box + m_\omega^2\right)\omega^\mu = g_\omega\bar{\psi}\gamma^\mu\psi - c_3\left(\omega_\mu\omega^\mu\right)\omega^\mu$$

$$\left(\Box + m_\sigma^2\right)\varphi = -g_s\bar{\psi}\psi - g_2\varphi^2 - g_3\varphi^3$$

$$\gamma_\mu\left(\mathrm{i}\partial^\mu + g_\omega\omega^\mu + g_\rho\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu + \left(M + g_s\varphi\right)\right)\psi = 0.$$

Field theories are difficult to solve. But nuclear systems are dense ones, one can neglect fluctuations and use a semiclassical approximation: Substitute *source-current* terms by their expectation values:

$$
\begin{aligned}
\bar{\psi}\psi &\longrightarrow \langle \bar{\psi}\psi \rangle \\
\bar{\psi}\gamma^{\mu}\psi &\longrightarrow \langle \bar{\psi}\gamma^{\mu}\psi \rangle \\
\bar{\psi}\boldsymbol{\tau}\gamma^{\mu}\psi &\longrightarrow \langle \bar{\psi}\boldsymbol{\tau}\gamma^{\mu}\psi \rangle
\end{aligned}
$$

Also substitute *meson fields* by their expectation values:

$$
\begin{aligned}
\varphi &\longrightarrow \langle \varphi \rangle \\
\omega^{\mu} &\longrightarrow \langle \omega^{\mu} \rangle \\
\boldsymbol{\rho}^{\mu} &\longrightarrow \langle \boldsymbol{\rho}^{\mu} \rangle .
\end{aligned}
$$

$$
\begin{aligned}
\varphi_0 \equiv \langle \varphi \rangle &= -\frac{g_s}{m_\sigma^2}\langle \bar{\psi}\psi \rangle - \frac{1}{m_\sigma^2}\left(g_2\varphi_0^2 + g_3\varphi_0^3\right) \\
\omega_0 \equiv \langle \omega^0 \rangle &= \frac{g_\omega}{m_\omega^2}\langle \bar{\psi}\gamma^0\psi \rangle - \frac{c_3}{m_\omega^2}\omega_0^3 \\
\rho_0 \equiv \langle \rho_3^0 \rangle &= \frac{g_\rho}{m_\rho^2}\langle \bar{\psi}\tau_3\gamma^0\psi \rangle .
\end{aligned}
$$

In the mean field style, one expands the multi-particle state into a product of single-particle states ψ_α. In the simplest approach (Hartree), the product is not antisymmetrized

$$
\begin{aligned}
\langle \bar{\psi}\psi \rangle &= \sum_\alpha a_\alpha \bar{\psi}_\alpha \psi_\alpha \\
\langle \bar{\psi}\gamma^0\psi \rangle &= \sum_\alpha a_\alpha \bar{\psi}_\alpha \gamma^0 \psi_\alpha \\
\langle \bar{\psi}\tau_3\gamma^0\psi \rangle &= \sum_\alpha a_\alpha \bar{\psi}_\alpha \tau_3 \gamma^0 \psi_\alpha .
\end{aligned}
$$

And the result is a Dirac equation for each single-nucleon state ψ_α

$$\left(-\mathrm{i}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}+\beta\left(M+g_s\varphi_0\right)+g_\omega\omega_0+g_\rho\tau_3\rho_0\right)\psi_\alpha=E_\alpha\psi_\alpha .$$

And many different versions of the lagrangians have been cooked

	Wa [19]	HS [8]	NL1 [15]	NL-SH [22]	TM1 [23]	TM2 [23]
M (MeV)	938.0	939.0	938.0	939.0	938.0	938.0
m_σ (MeV)	550.00	492.0	492.250	526.059	511.198	526.443
m_ω (MeV)	783.00	783.0	795.359	783.0	783.0	783.0
m_ρ (MeV)	763.00	770.0	763.0	763.0	770.0	770.0
g_s	9.58289	10.47	10.1377	10.444	10.0289	11.4694
g_ω	11.683586	13.80	13.2846	12.945	12.6139	14.6377
g_ρ	0.0	8.07	4.9757	4.383	4.6322	4.6783
g_2 (fm^{-1})	0.0	0.0	-12.1724	-6.9099	-7.2325	-4.4440
g_3	0.0	0.0	-36.2646	-15.8337	0.6183	4.6076
c_3	0.0	0.0	0.0	0.0	71.3075	84.5318
E/A (MeV)	-15.70	-15.70	-16.40	-16.32	-18.56	-14.22
ρ_B (fm^{-1})	-0.193	0.148	0.152	0.146	0.146	0.111
M^*/M	0.55	0.54	0.573	0.597	0.66	0.618

Relativistic mean field RMF

- Use Dirac equation with local potentials, obtained with a lagrangian fitted to reproduce saturation properties of nuclear matter, and/or radii and mass of selected nuclei. Or use any phenomenological S-V potentials of Woods-Saxon kind

- RMF does saturate, even if no Fock terms are introduced (Dirac Hartree) and without considering correlations

- Generally speaking, introducing Fock terms or correlations shifts the saturation point, but a change of the parameters of the model puts the saturation point wherever we want it

- This is at serious variance with the nonrelativistic case. Are there any observables sensitive to these differences?

•At nuclear saturation density 0.16 fm^{-3}, the empirical fields deduced from fits to finite nuclei or nuclear matter are of the order of 300 to 500 MeV

•The single particle potential in which the nucleons move originates from the cancellation of the two contributions yielding around −50 MeV which makes it difficult to observe relativistic effects in nuclear systems

•There exist, however, several features in nuclear structure which can naturally be explained within Dirac phenomenology while models based on non-relativistic dynamics have difficulties:

•Best established is the large spin-orbit splitting in finite nuclei

•Also the so-called pseudo-spin symmetry, observed more than thirty years ago in single particle levels of spherical nuclei, can naturally be understood within RMF theory as a consequence of the coupling to the lower components of the Dirac equation

•QCD sum rules also suggest attractive scalar and repulsive vector self-energies which are astonishingly close to the empirical values derived from RMF fits to the nuclear chart

•Also relativistic many-body calculations [6, 7, 8] yield scalar/vector fields of the same sign and magnitude as obtained from RMF theory or, alternatively, from QCD sum rules

These facts suggest that preconditions for the existence of large fields in matter or, alternatively, the density dependence of the QCD condensates, must already be inherent in the vacuum nucleon-nucleon (NN) interaction

If this is true, then a simple, phenomenological RMF would be a very effective way of emulating the underlying microscopic relativistic strongly interacting theory

Local potentials?

$$(\alpha \cdot \mathbf{p} + \beta (M + U_s) - E + U_\omega)\,\psi(\mathbf{x}) = 0.$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}.$$

$$\left[\frac{1}{2m}\vec{\mathrm{p}}^{\,2} + U_e(r;E) + U_{\mathrm{so}}(r)\frac{\vec{\sigma}\cdot\vec{\mathrm{L}}}{r}\right]\psi_>(\vec{\mathrm{r}}) = \frac{1}{2m}(E^2 - m^2)\psi_>(\vec{\mathrm{r}}),$$

$$U_{\mathrm{so}}(r) \approx (2m)^{-2}\frac{d}{dr}[U_0(r) - U_s(r)],$$

$$U_e(r;E) \approx U_s(r) + Em^{-1}U_0(r) + (2m)^{-1}[U_s^{\,2}(r) - U_0^{\,2}(r)].$$

Relativistic mean field RMF (II)

•By chosing the parameters of the lagrangians to reproduce the saturation point at the mean field approximation, the effects of correlations on the saturation curve has been taken into account, at least partly

•The relativistic mean field, being able of incorporating at the same time repulsive and attractive effects, via vector and scalar potentials, should be more succesful in emulating correlations

• Nonlocalities (dependences of the potentials or the effective mass on the density, the energy or the position) as well as other effects introduced by correlations or even Fock terms, will be recovered from the relativistic formalisms when performing the nonrelativistic reduction, even if the relativistic equations and potentials are local

•Of course, RMF also incorporates relativistic effects!!