Green Function Formalism and Electroweak Nuclear Response

Lecture 4

Omar Benhar

INFN and Department of Physics

Università “La Sapienza”, I-00185 Roma

Neutrino-nucleus scattering: (obvious) motivation

- ⋆ Neutrino experiments use nuclei as detectors
- ⋆ Quantitative understanding of the weak nuclear response at $E_\nu \sim 0.5 - 3$ GeV required for data analysis
- ⋆ Need to develop a theoretical approach
  - ▷ testable against electron scattering data
  - ▷ applicable to a wide range of kinematical conditions and targets
  - ▷ easily implementable in Monte Carlo simulations

Charged current neutrino-nucleus scattering in the IA regime

⋆ Cross section of the process $\nu_\ell + A \to \ell^- + X$

$$\frac{d\sigma_A}{d\Omega_{\ell'} dE_{\ell'}} = \int d^4p \, P(p) \, \left(\frac{d\sigma_N}{d\Omega_{\ell'} dE_{\ell'}} \right)$$

Charged current neutrino-nucleus scattering in the IA regime

⋆ Cross section of the process $\nu_\ell + A \to \ell^- + X$

$$\frac{d\sigma_A}{d\Omega_{\ell'} dE_{\ell'}} = \int d^4p \, P(p) \left(\frac{d\sigma_N}{d\Omega_{\ell'} dE_{\ell'}} \right)$$

⋆ Elementary cross-section

$$\frac{d^2\sigma_N}{d\Omega_{\ell'} dE_{\ell'}} = \frac{G_F^2 \, V_{ud}^2}{32\,\pi^2} \, \frac{|\mathbf{k}'|}{|\mathbf{k}|} \, \frac{1}{4\,E_{\mathbf{p}}\,E_{|\mathbf{p}+\mathbf{q}|}} \, L_{\mu\nu} W^{\mu\nu} \, .$$

$$W^{\mu\nu} = -g^{\mu\nu}\,W_1 + p^\mu\,p^\nu\,\frac{W_2}{m_N^2} + i\,\varepsilon^{\mu\nu\alpha\beta}\,\widetilde{q}_\alpha\,p_\beta\,\frac{W_3}{m_N^2}$$

$$+\widetilde{q}^\mu\,\widetilde{q}^\nu\,\frac{W_4}{m_N^2} + (p^\mu\,\widetilde{q}^\nu + p^\nu\,\widetilde{q}^\mu)\,\frac{W_5}{m_N^2}$$

Nucleon weak structure functions

⋆ In the case of QE scattering the W_i are related to the form factors through

$$W_1 = 2\left[-\frac{\widetilde{q}^2}{2}\,(F_1+F_2)^2 + \left(2\,m_N^2 - \frac{\widetilde{q}^2}{2}\right)\,F_A^2\right]\delta(\tilde{s}-m_N^2)$$

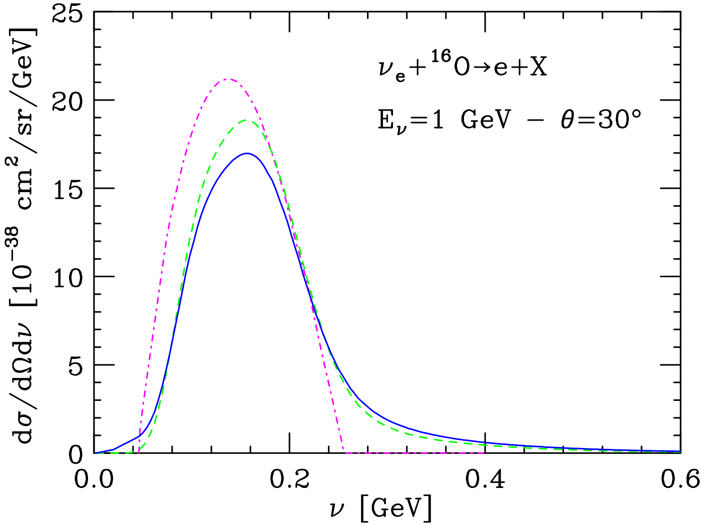
$$W_2 = 4\left[F_1^2 - \left(\frac{\widetilde{q}^2}{4\,m_N^2}\right)\,F_2^2 + F_A^2\right]\delta(\tilde{s}-m_N^2)$$

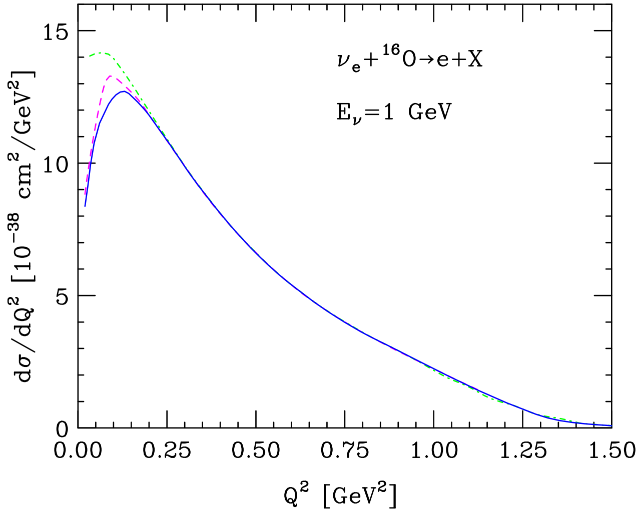
$$W_3 = -4\,(F_1+F_2)\,F_A\delta(\tilde{s}-m_N^2)$$

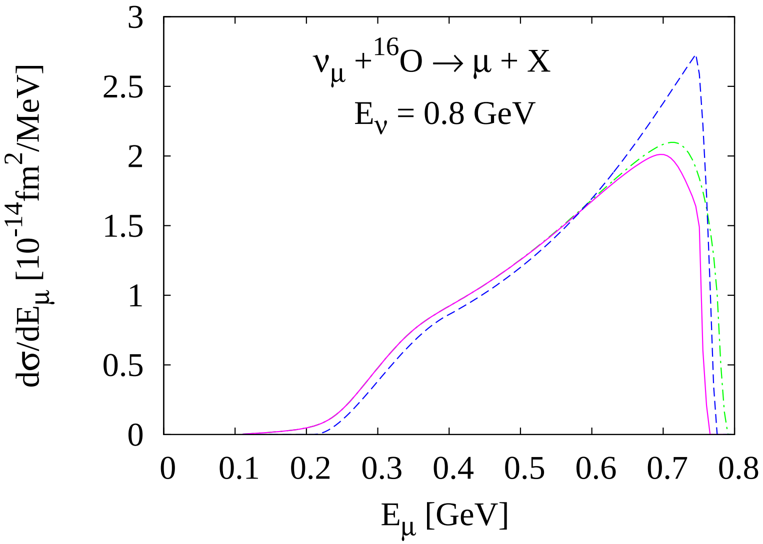
$$W_4 = -2\left[F_1\,F_2 + \left(2\,m_N^2 + \frac{\widetilde{q}^2}{2}\right)\frac{F_2^2}{4\,m_N^2} + \frac{\widetilde{q}^2}{2}\,F_P^2 - 2\,m_N\,F_P\,F_A\right]\delta(\tilde{s}-m_N^2)$$

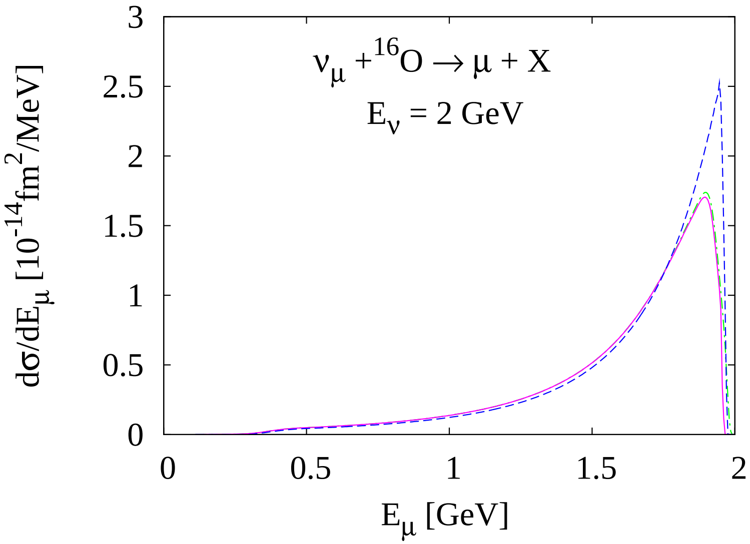
$$W_5 = \frac{W_2}{2}$$

Results for $^{16}O\ (\nu_e, e)$ scattering

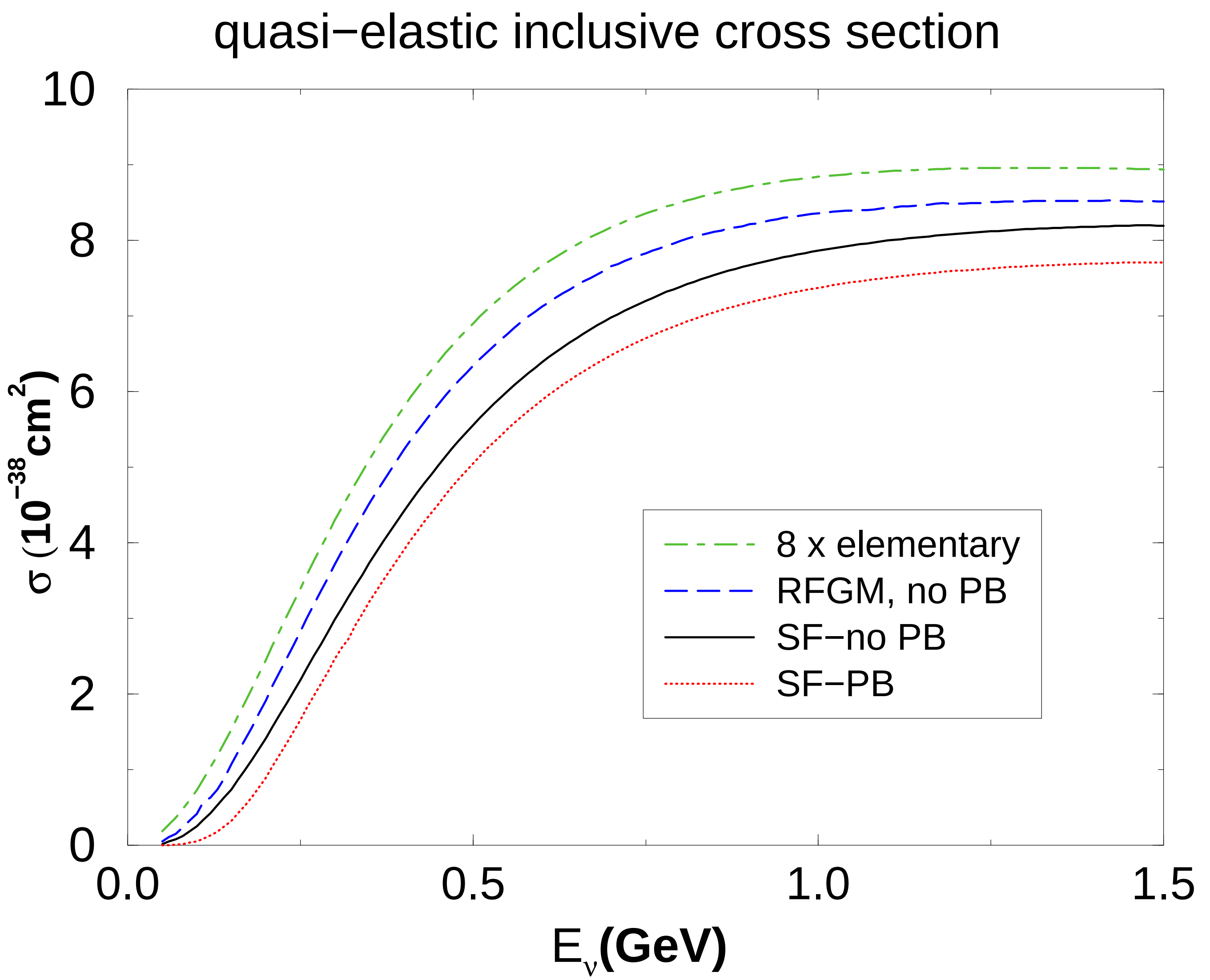






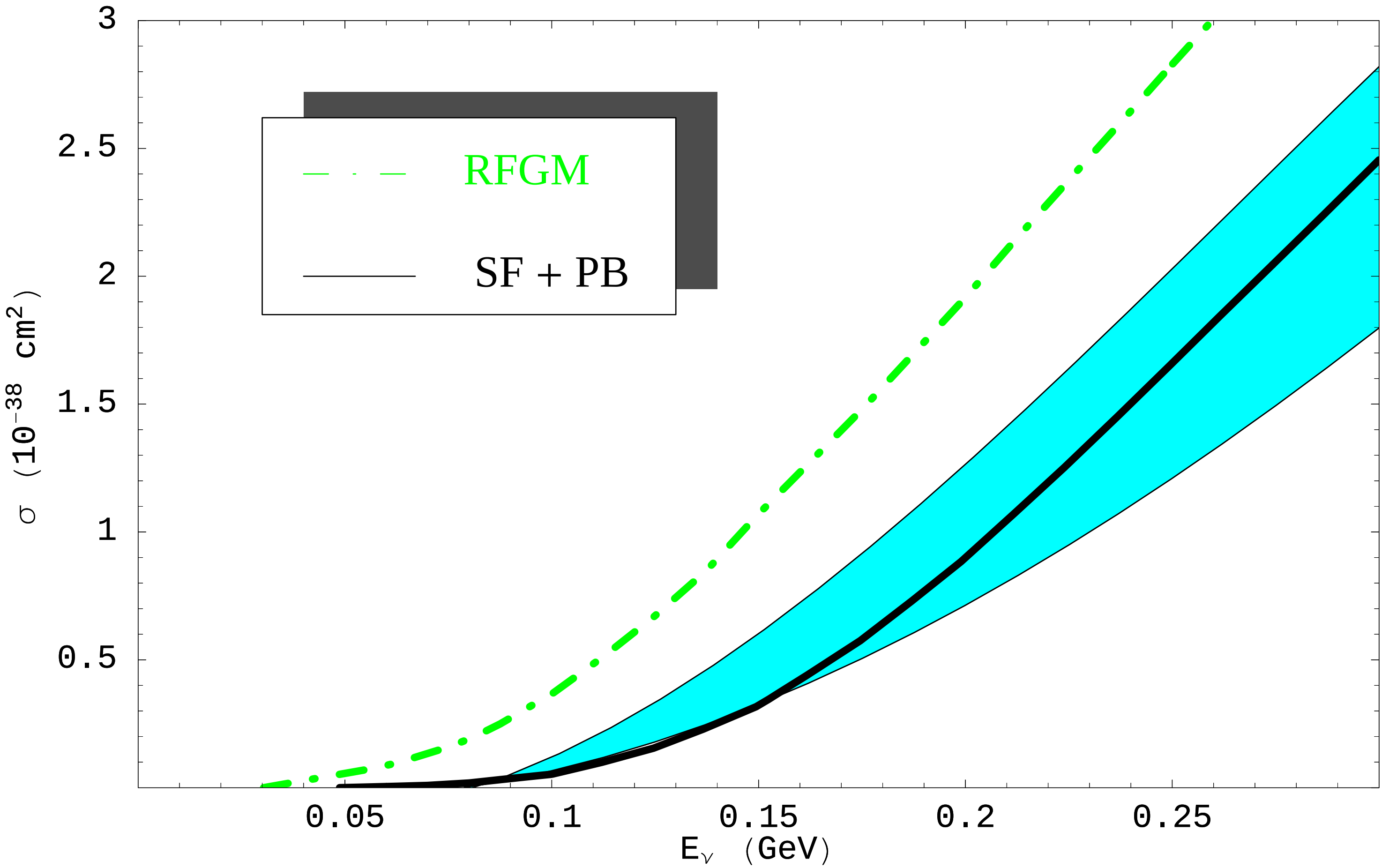


Total x-section $\sigma(\nu_e +^{16} O \rightarrow e^- + X)$



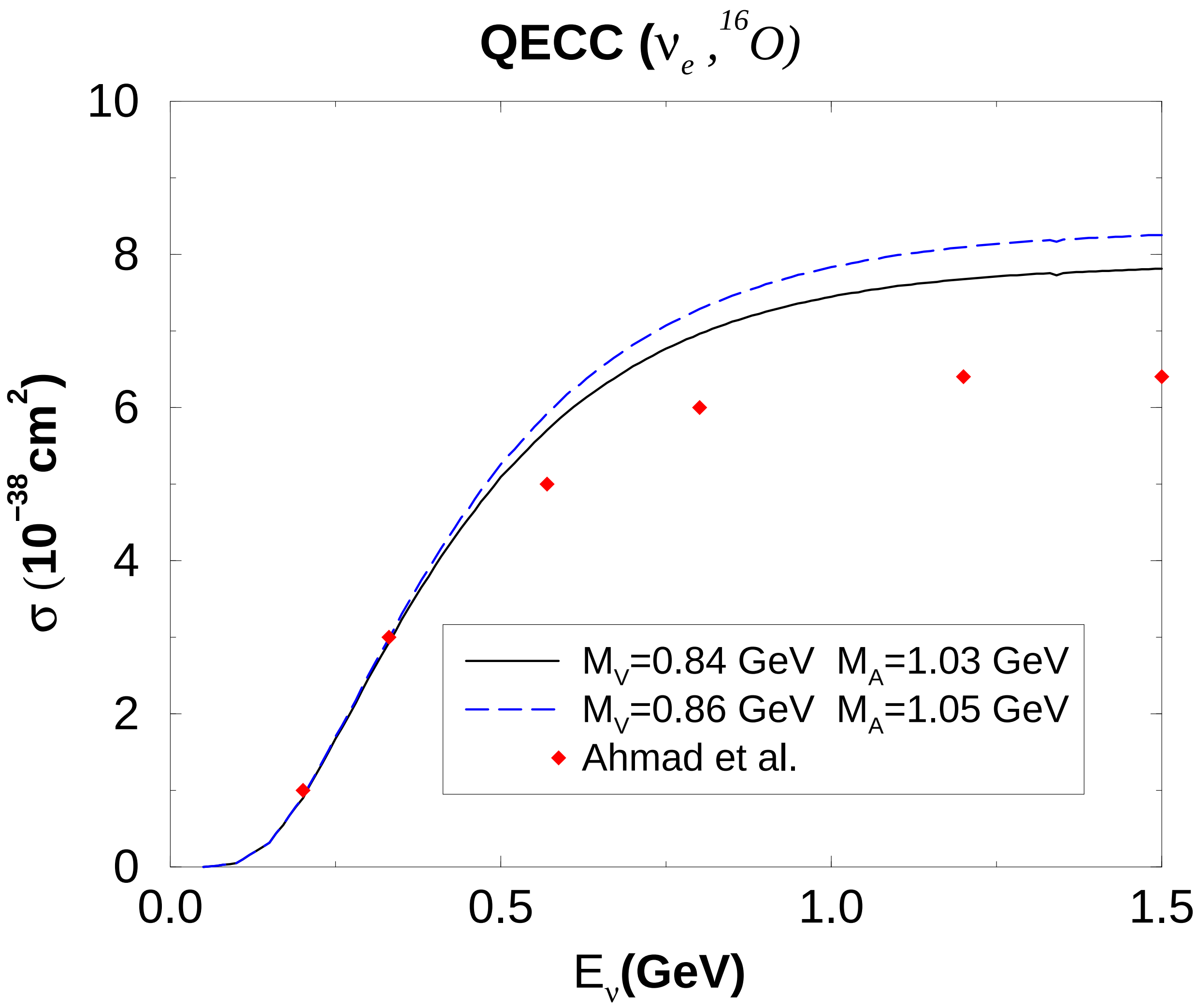
$\sigma(\nu_e + ^{16}O \to e^- + X)$

- ⋆ Comparison to the results of Amaro, Nieves & Valverde



$\sigma(\nu_e +^{16} O \rightarrow e^- + X)$

- Comparison to the results of Ahmad, Sajjad Athar & Singh



π-production through Δ excitation

- the contribution of the processes

$$\nu_\ell + n \to \ell + \Delta^+ \quad , \quad \nu + p \to \ell + \Delta^{++}$$

can be readily included using the same formalism

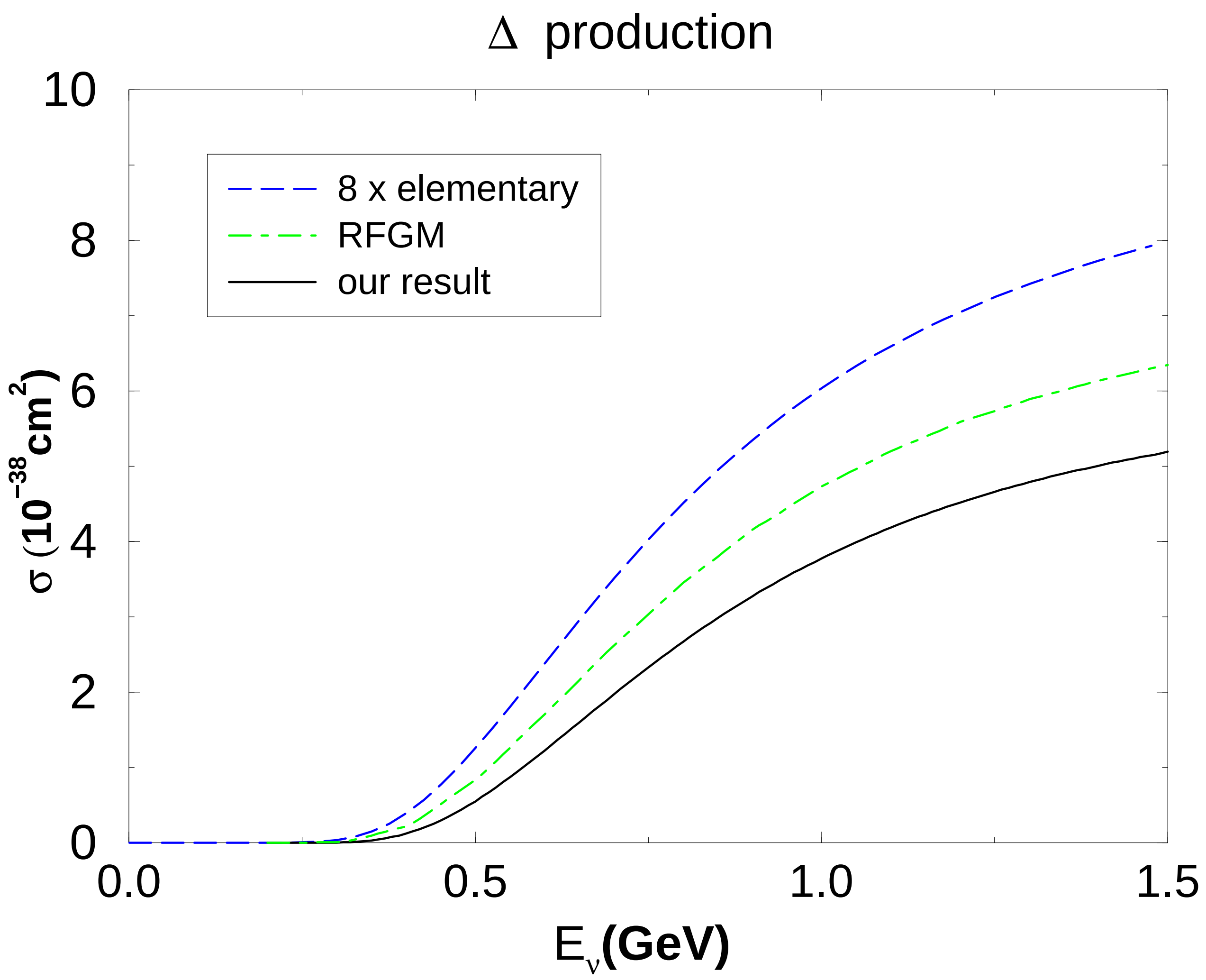
- Replace the energy conservig δ-function in W_i with

$$\frac{M_R \Gamma_R}{\pi} \, \frac{1}{(W^2 - M_R^2)^2 + M_R^2 \Gamma_R^2}$$

- Form factors from the model of Lalakulich & Paschos . Use isospin symmetry to relate Δ^{++} and Δ^+ form factors through

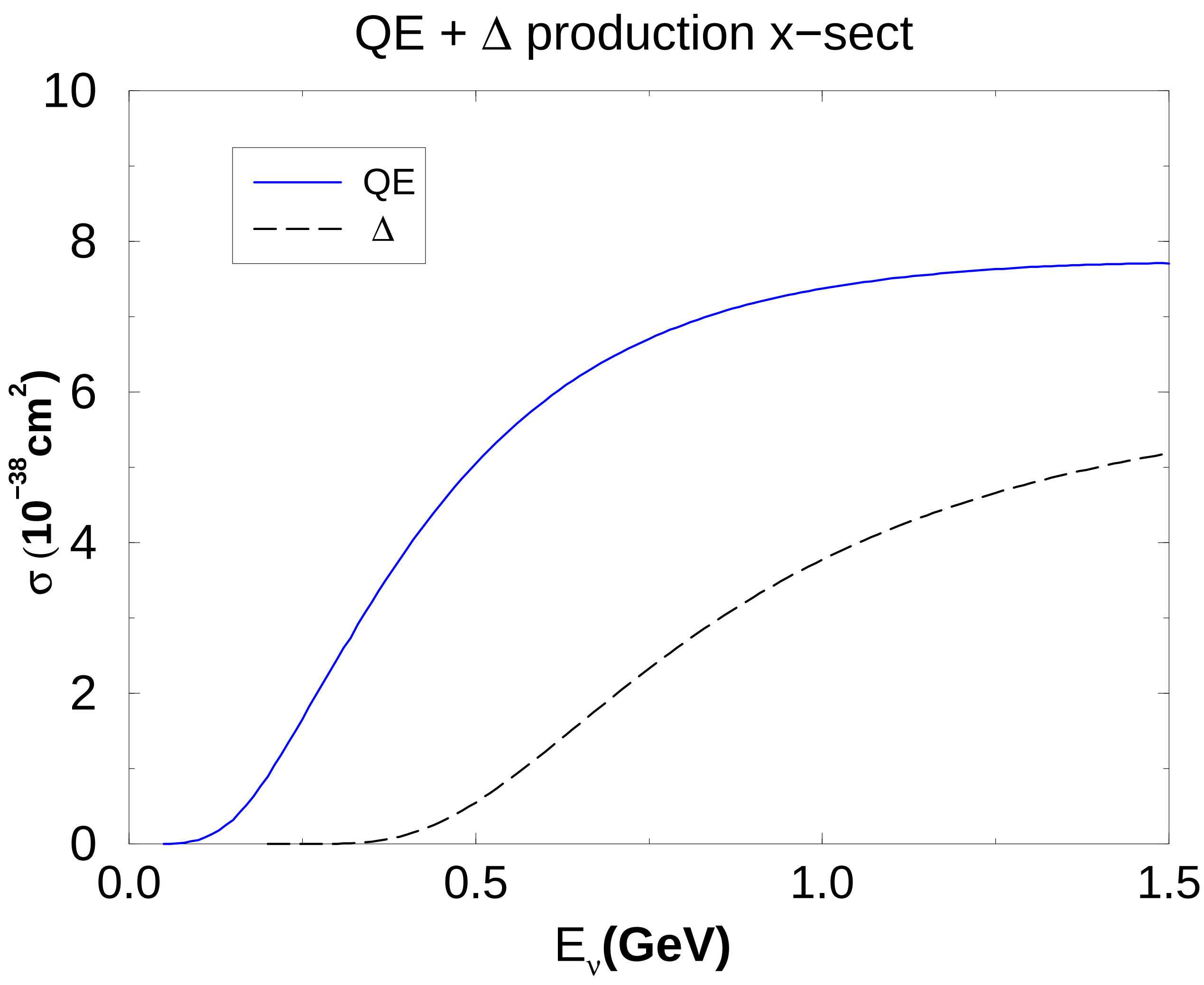
$$\langle \Delta^{++} | J_\mu^A | p \rangle = \sqrt{3} \, \langle \Delta^+ | J_\mu^A | n \rangle \ .$$

Δ production in $\nu_e +^{16} O \rightarrow e^- + X$



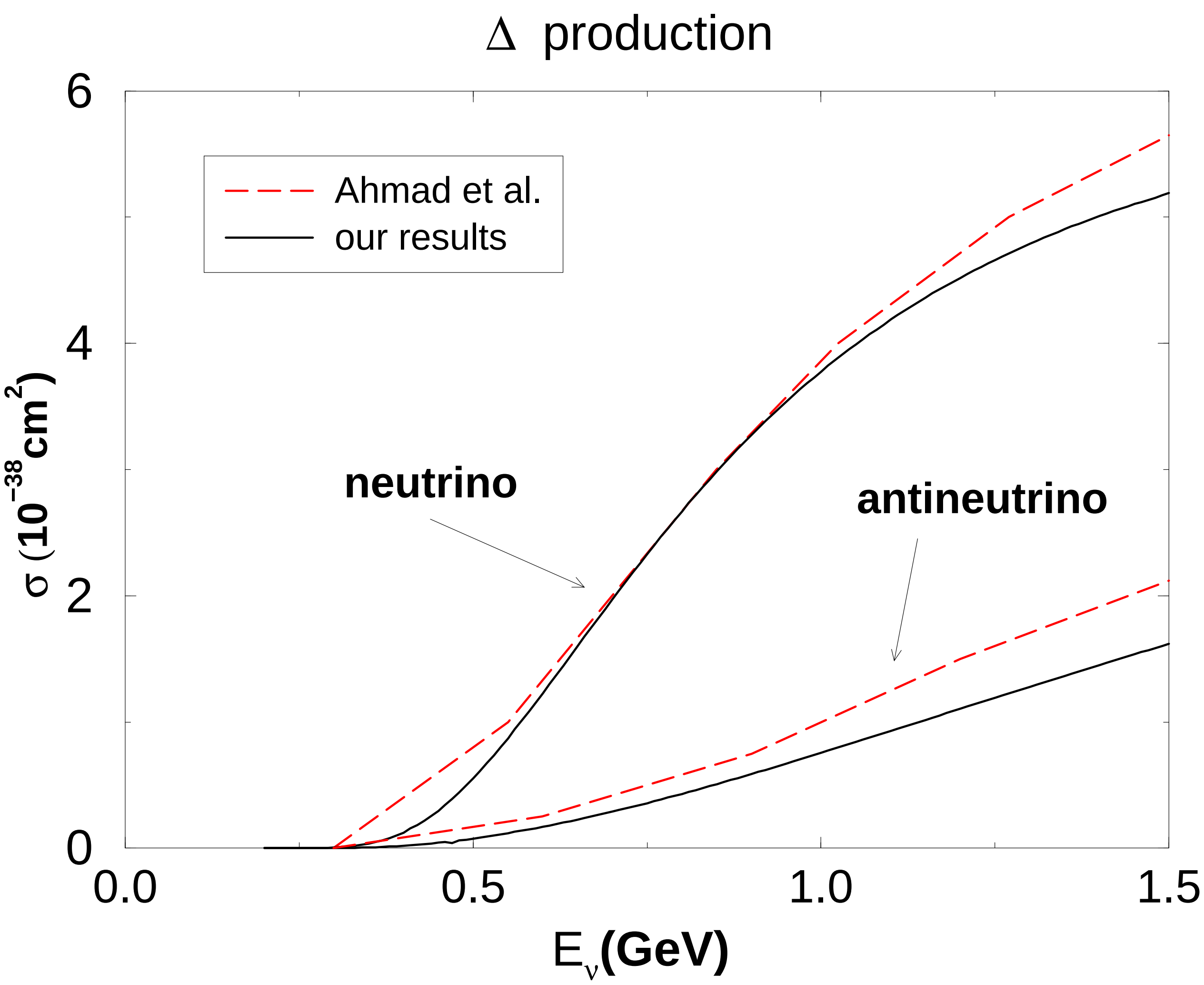
Δ production in $\nu_e +^{16} O \to e^- + X$ (continued)

- Comparison to QE



Δ production in $\nu_e + {}^{16}O \to e^- + X$ (continued)

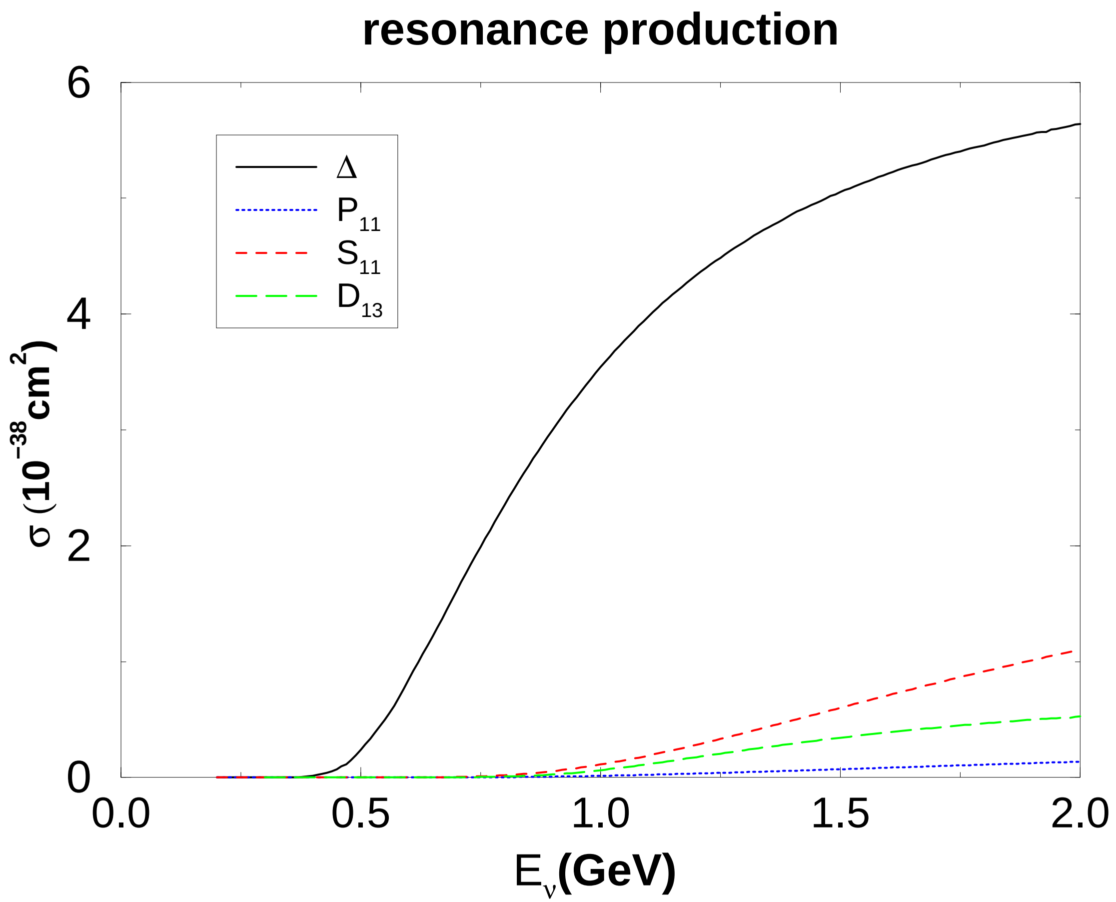
- Comparison to the results of Ahmad, Sajjad Athar & Singh



Production of higher resonances

- Inclusion of the three isospin 1/2 states (Form factors from Lalakulich, Paschos & Piranishvili).

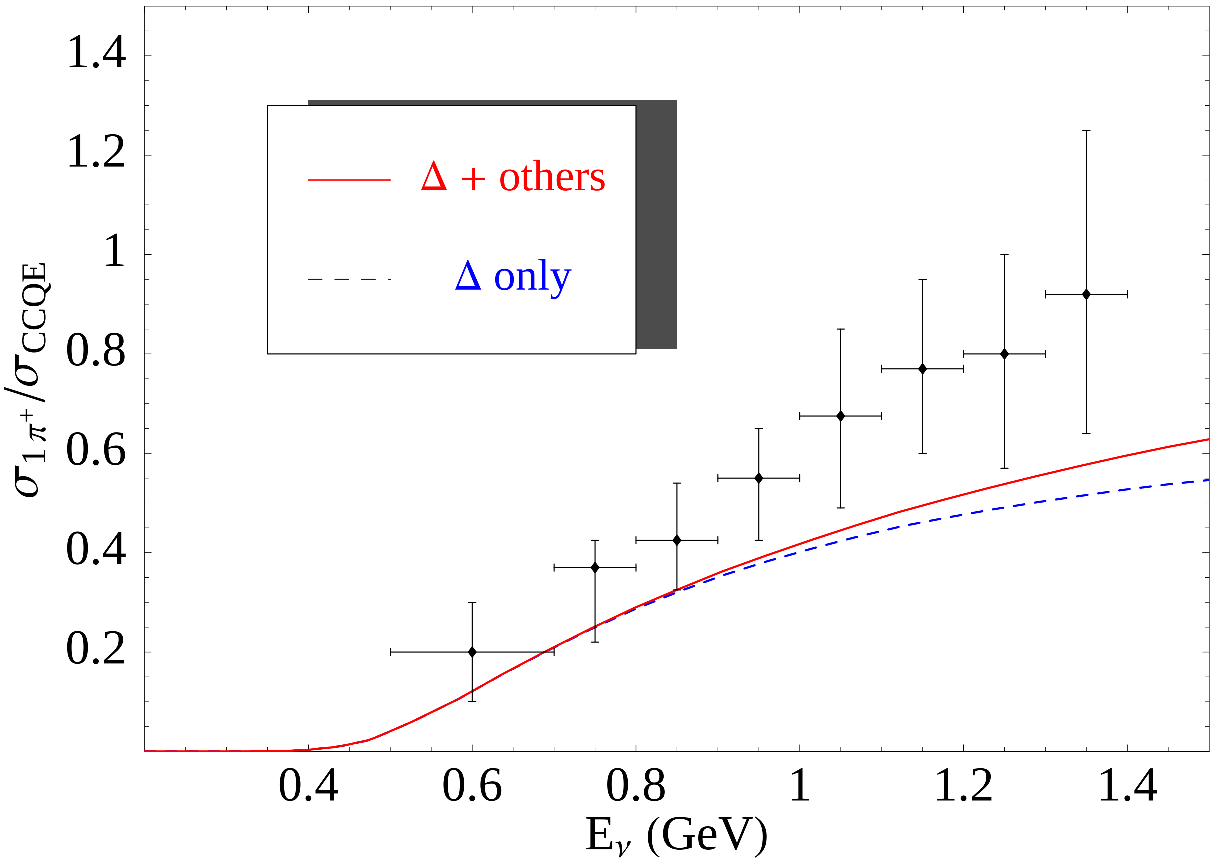
$$P_{11}(1440) \quad , \quad D_{13}(1520) \quad , \quad S_{11}(1535)$$



Comparison to π^+ production (preliminary) data

$$R = \frac{\sigma_{\pi^+}}{\sigma_{CCQE}} \;\;,\;\; \sigma_{\pi^+} = \frac{10}{9}\,\sigma_{\Delta^{++}} + \frac{8}{9}\,(b_1\,\sigma_{P_{11}} + b_2\,\sigma_{D_{13}} + b_3\,\sigma_{S_{11}})$$

- ⋆ M. O. Wascko (MiniBooNE) arXiv:hep-ex/0602050



Extracting nucleon properties from nuclear x-section

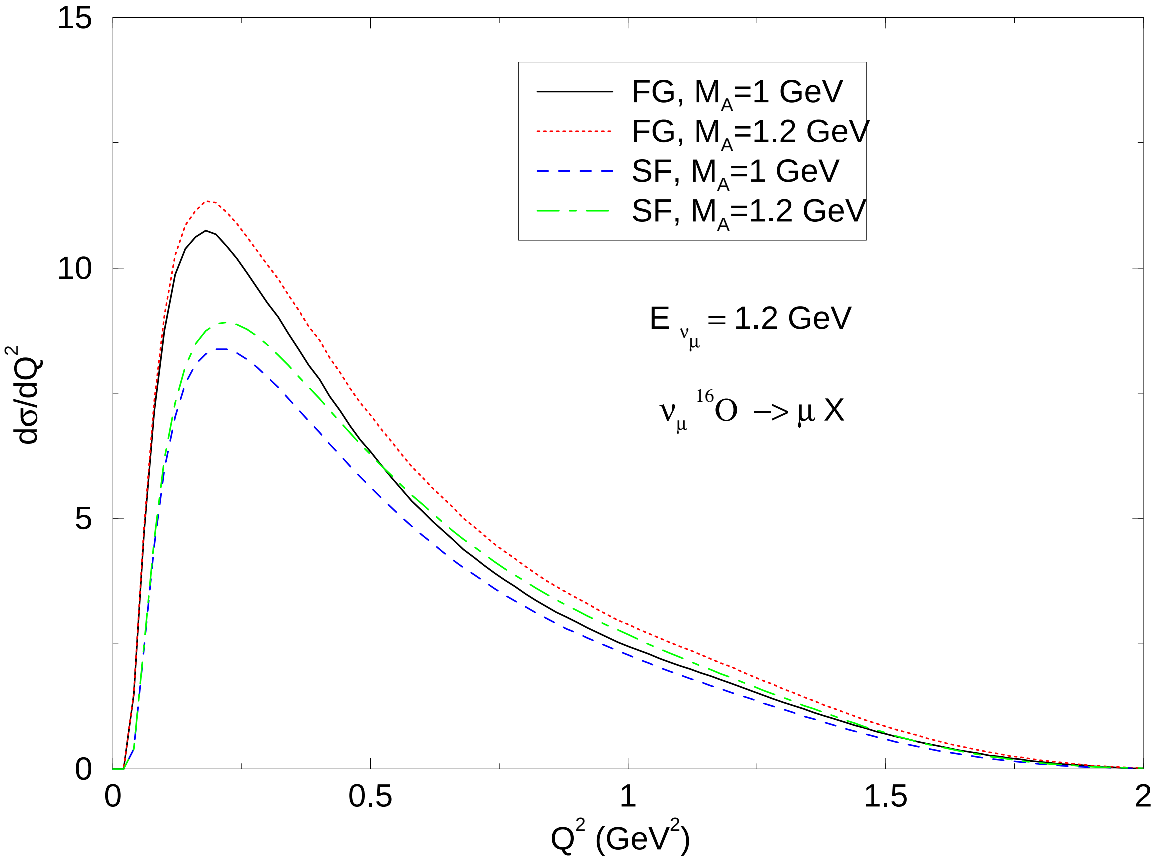
- ⋆ K2K and MiniBooNE have reported a value of the nucleon axial mass, $M_A \sim 1.2$, larger than that previously determined from deuterium data, extracted from the analysis of neutrino scattering off oxygen and carbon

Extracting nucleon properties from nuclear x-section

- ⋆ K2K and MiniBooNE have reported a value of the nucleon axial mass, $M_A \sim 1.2$, larger than that previously determined from deuterium data, extracted from the analysis of neutrino scattering off oxygen and carbon

- ⋆ The authors of the MiniBooNE paper state that "The MA value reported here should be considered an *effective parameter* in the sense that it may be incorporating nuclear effects not otherwise included in the RFG model. In particular, it may be that a more proper treatment of the nucleon momentum distribution in the RFG would yield an MA value in closer agreement to that measured on deuterium."

⋆ Effect of the axial mass on the Q-distribution at fixed E_ν



⋆ Using a realistic spectral function and increasing M_A leads to changes of opposite sign

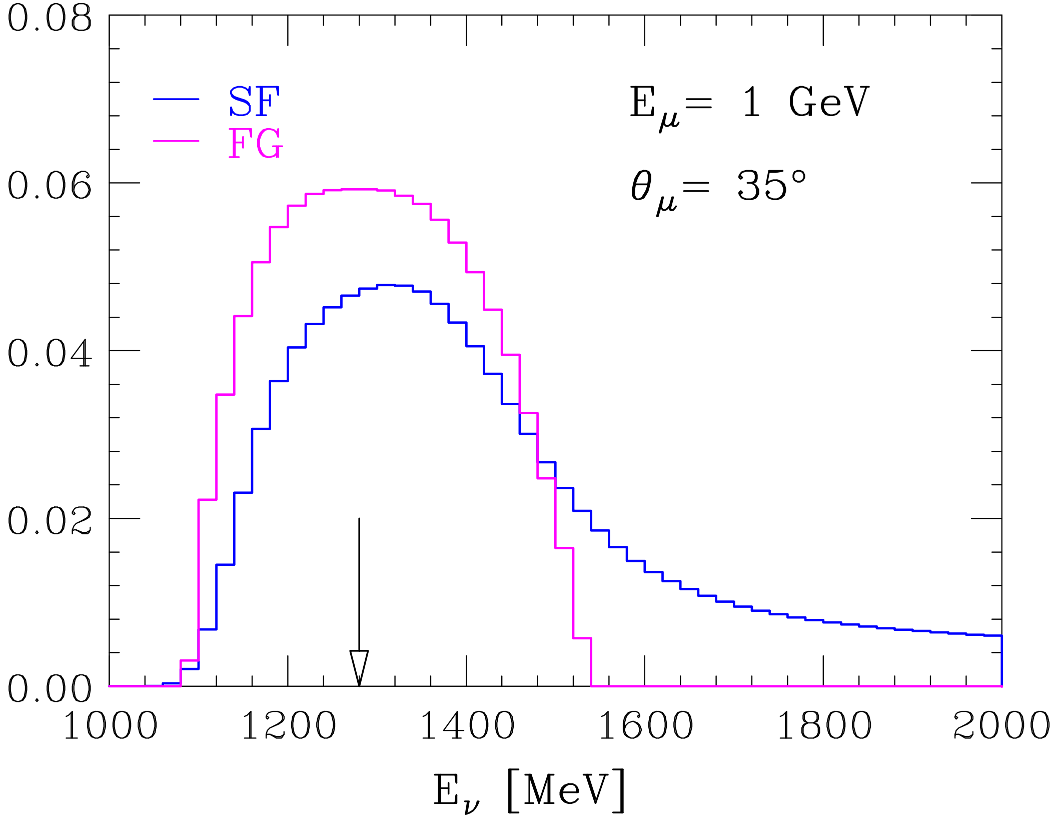
- To what extent is the determination of M_A from nuclear data biased by the treatment of nuclear effects in the analysis ?
- Main assumptions of the K2K and MiniBooNE analysis
  - Fermi gas model
  - Reconstructed neutrino energy obtained assuming that the neutrino hit a stationary neutron bound with constant energy $\epsilon \sim 25 \div 35$ MeV
- Under these assumptions, at fixed E_μ and θ_μ, from

$$(k_\nu + p_n - k_\mu)^2 = m_p^2$$

it follows that ($\Delta m^2 = m_n^2 - m_p^2$)

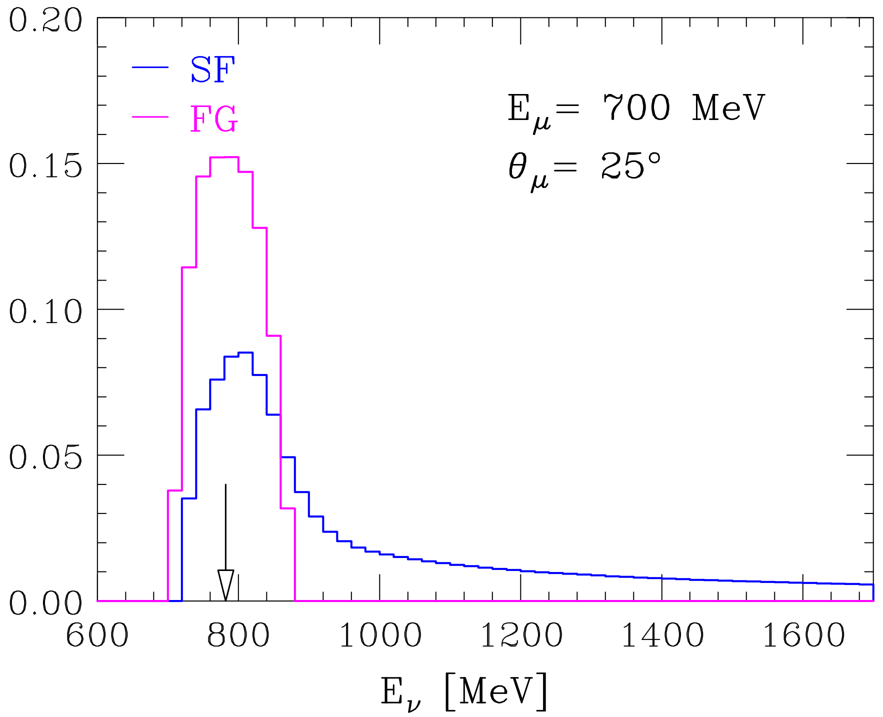
$$E_\nu = \frac{2(m_n - \epsilon)E_\mu - (\epsilon^2 - 2m_n\epsilon + m_\mu^2 + \Delta m^2)}{2(m_n - \epsilon - E_\mu + |\mathbf{k}_\mu| \cos\theta_\mu)} \ ,$$

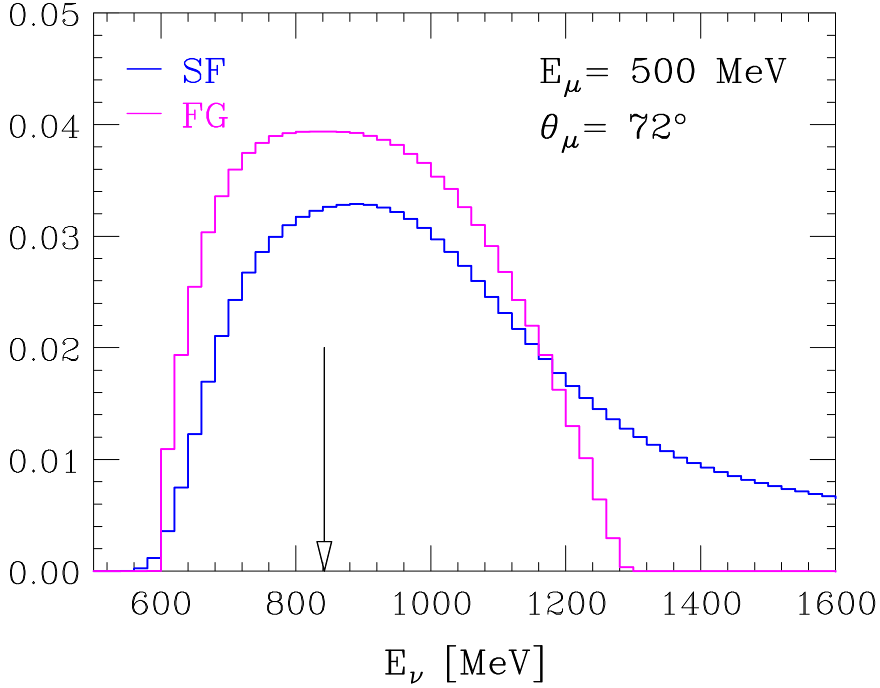
⋆ Releasing these approximations the distribution of neutrino energy can be reconstructed solving the kinematic equation with neutron momentum and energies sampled from the probability distribution associated with the spectral function



⋆ Note: the two histograms have the same normalization

Effects of NN interactions on the nuclear response (continued)





Conclusions (quoting Vijay R. Pandharipande)

- ⋆ Approximations are often needed. They are certainly necessary in the description of nuclear dynamics

Conclusions (quoting Vijay R. Pandharipande)

- ⋆ Approximations are often needed. They are certainly necessary in the description of nuclear dynamics
- ⋆ Approximations can be good or bad. This is often a highly contoversial subject

Conclusions (quoting Vijay R. Pandharipande)

- ⋆ Approximations are often needed. They are certainly necessary in the description of nuclear dynamics
- ⋆ Approximations can be good or bad. This is often a highly contoversial subject
- ⋆ Unnecessary approximations should never be used !