Green Function Formalism
and
Electroweak Nuclear Response

Lecture 3

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Outline

- ⋆ Electron-nucleus scattering

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  - ▷ The inclusive electron-nucleus cross section

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  - ▷ Comparison to inclusive data: quasielastic and inelastic contributions to the nuclear cross section

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- Electron-nucleus scattering
  - The inclusive electron-nucleus cross section
  - The impulse approximation regime
  - Comparison to inclusive data: quasielastic and inelastic contributions to the nuclear cross section
  - Extension to semiexclusive and exclusive processes

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- ⋆ All the information on target structure is contained in $W^{\mu\nu}$ (reminiscent of $S(\mathbf{q}, \omega)$ of the previous Lectures)

$$W^{\mu\nu} = \sum_X \langle 0 | J^\mu | X \rangle \langle X | J^\nu | 0 \rangle \delta^{(4)}(p_0 + q - p_X)$$

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  - ▷ final state particles carry large momenta ($\sim \mathbf{q}$)
  - ▷ the em interaction may be *inelastic*, leading to the appearance of hadrons other than protons and neutrons

⋆ Lorentz covariance, gauge invariance and conservation of parity require

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{W_2}{M^2} \left(p_0^\mu - \frac{(p_0 q)}{q^2} q^\mu \right) \left(p_0^\nu - \frac{(p_0 q)}{q^2} q^\nu \right) \ ,$$

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⋆ The cross section in the Lab frame

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- $\star$ Rewrite in terms of *longitudinal* and *transverse* response functions

$$\frac{d^2\sigma}{d\Omega_{e'} dE_{e'}} = \left(\frac{d\sigma}{d\Omega_{e'}} \right)_M \left[\frac{Q^2}{|\mathbf{q}|^2} R_L(|\mathbf{q}|, \omega) \left(\frac{1}{2} \frac{Q^2}{|\mathbf{q}|^2} + \tan^2 \frac{\theta}{2} \right) R_T(|\mathbf{q}|, \omega) \right] ,$$

with

$$R_T(|\mathbf{q}|, \omega) = 2W_1(|\mathbf{q}|, \omega) \ , \ \frac{Q^2}{|\mathbf{q}|^2} R_L(|\mathbf{q}|, \omega) = W_2(|\mathbf{q}|, \omega) - \frac{Q^2}{|\mathbf{q}|^2} W_1(|\mathbf{q}|, \omega)$$

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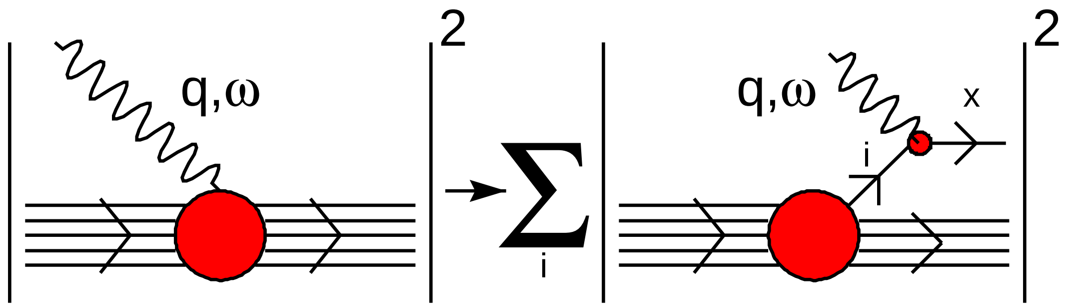

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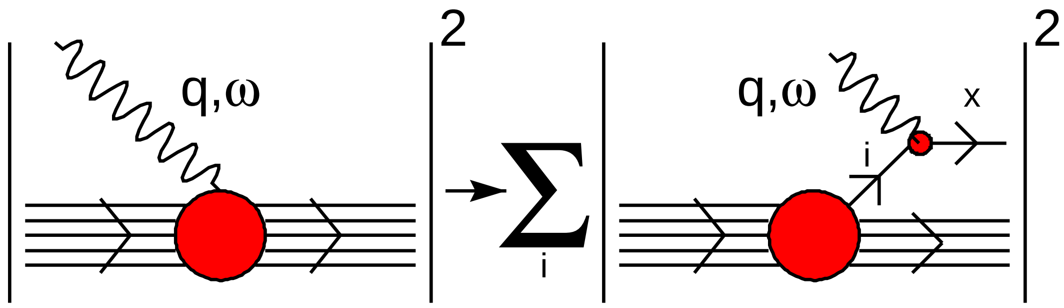

- replace *nuclear* current with the sum of individual *nucleon* currents

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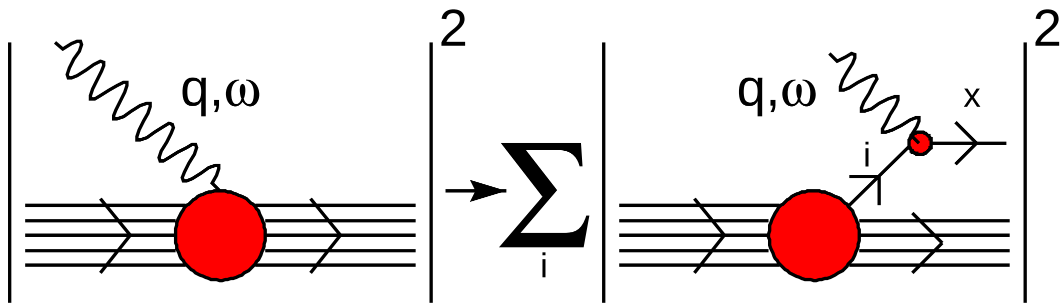

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▷ write the final state in the factorized form (note: $|x, \mathbf{p}_x\rangle$ can be any hadronic final state)

$$|X\rangle \to |x, \mathbf{p}_x\rangle \otimes |\mathcal{R}, \mathbf{p}_\mathcal{R}\rangle \ .$$

⋆ Within IA

$$\langle 0|J^\mu|X\rangle = \frac{m}{\sqrt{|\mathbf{p}_\mathcal{R}|^2+m^2}}\langle 0|\mathcal{R},\mathbf{p}_\mathcal{R};\mathrm{N},-\mathbf{p}_\mathcal{R}\rangle \sum_i \langle -\mathbf{p}_\mathcal{R},N|j_i^\mu|x,\mathbf{p}_x\rangle$$

where $|\mathrm{N},\mathbf{k}\rangle$ is the state describing a *free* nucleon carrying momentum $\mathbf{k}$

$$W^{\mu\nu}(\mathbf{q},\omega) = \int d^3k\, dE\, \left(\frac{m}{E_\mathbf{k}}\right)\left[ZP_p(\mathbf{k},E)w_p^{\mu\nu}(\widetilde{q}) + NP_n(\mathbf{k},E)w_n^{\mu\nu}(\widetilde{q})\right]$$

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⋆ The IA x-section is determined by

- ▷ The proton and neutron hole spectral functions (see Lecture 1)
- ▷ The em tensor describing a *free* nucleon carrying momentum **k**

$$w_N^{\mu\nu} = \sum_x\langle\mathbf{k},\mathrm{N}|j_N^\mu|x,\mathbf{k}+\mathbf{q}\rangle\langle\mathbf{k}+\mathbf{q},x|j_N^\nu|\mathrm{N},\mathbf{k}\rangle\delta(\widetilde{\omega}+E_\mathbf{k}-E_x)$$

$$\widetilde{\omega} = E_x - E_\mathbf{k} = E_0+\omega-E_\mathcal{R}-E_\mathbf{k} = \omega - E + m - E_\mathbf{k}$$

- ⋆ The replacement

$$q \equiv (\omega, \mathbf{q}) \to \widetilde{q} \equiv (\widetilde{\omega}, \mathbf{q})$$

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- ⋆ Note that the above replacement leads to a violation of current conservation, as

$$\widetilde{q}_\mu w_N^{\mu\nu} = 0 \quad , \quad q_\mu w_N^{\mu\nu} \neq 0$$

However, its effects become negligible at large $|\mathbf{q}|$

* The nucleon tensor can be written as

$$w_N^{\mu\nu} = w_1^N \left(-g^{\mu\nu} + \frac{\widetilde{q}^{\mu}\widetilde{q}^{\nu}}{\widetilde{q}^2} \right) + \frac{w_2^N}{m^2} \left(k^{\mu} - \frac{(k\widetilde{q})}{\widetilde{q}^2}\widetilde{q}^{\mu} \right) \left(k^{\nu} - \frac{(k\widetilde{q})}{\widetilde{q}^2}\widetilde{q}^{\nu} \right)$$

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★ In the case of quasielastic (QE) scattering

$$w_1^N = -\frac{\widetilde{q}^2}{4m^2}\, \delta\left(\widetilde{\omega} + \frac{\widetilde{q}^2}{2m} \right)\, G_{M_N}^2$$

$$w_2^N = \frac{1}{1 - \widetilde{q}^2/4m^2}\, \delta\left(\widetilde{\omega} + \frac{\widetilde{q}^2}{2m} \right) \left(G_{E_N}^2 - \frac{\widetilde{q}^2}{4m^2} G_{M_N}^2 \right)$$

where G_{E_N} and G_{M_N} are the nucleon electric and magnetic form factors

Local Density Approximation (LDA) $P(\mathrm{p}, E)$ for oxygen

$$P(\mathbf{p}, E) = P_{QP}(\mathbf{p}, E) + P_{corr}(\mathbf{p}, E)$$

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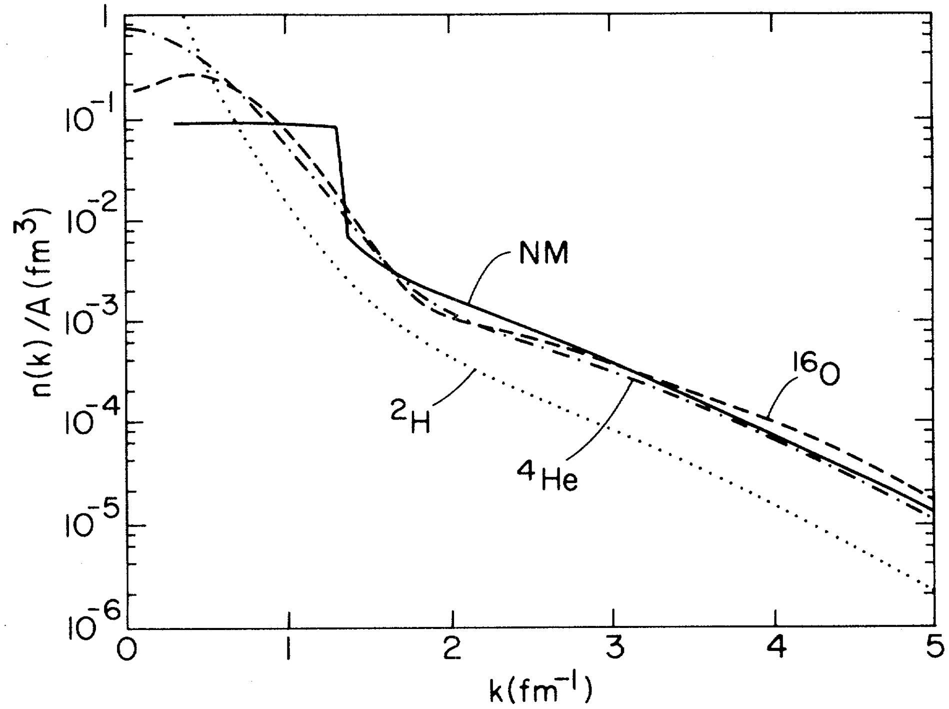
$$P_{corr}(\mathbf{p}, E) = \int d^3r \, \rho_A(r) \, P_{corr}^{NM}(\mathbf{p}, E; \rho = \rho_A(r))$$

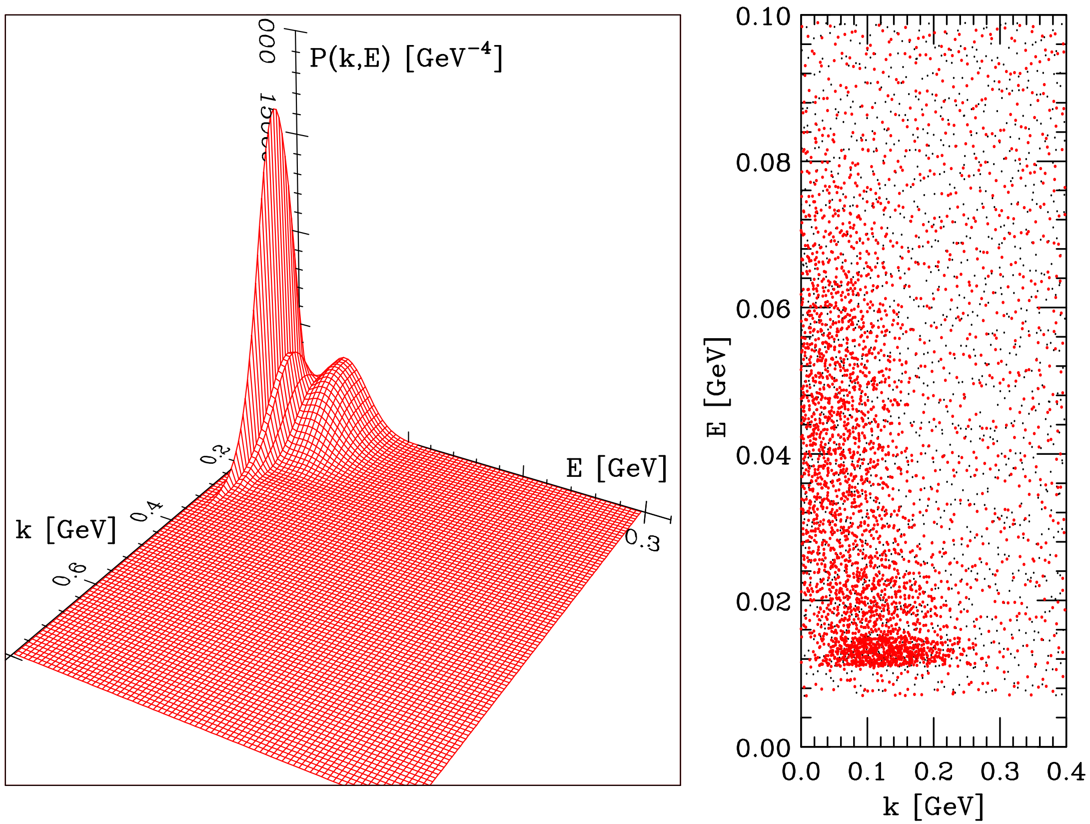
Why do we expect LDA to be OK ?



⋆ The momentum distribution

$$n(\mathbf{p}) = \int dE\, P(\mathbf{p}, E)$$

scales with A at $\mathbf{p} > p_F$ for $A > 2$



- ⋆ the shell model contribution $P_{QP}(\mathbf{p}, E)$ accounts for $\sim 80\%$ of the strenght
- ⋆ the remaining $\sim 20\%$, accounted for by $P_{corr}(\mathbf{p}, E)$, is located at high momentum *and* large removal energy

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  - energy shift due to the mean field of the spectators
  - redistributions of the strenght due to the coupling of $1p - 1h$ final state to $np - nh$ final states
- In *inclusive processes* at large momentum transfer FSI can be included through

$$\frac{d\sigma}{d\Omega_{e'} d\omega} = \int d\omega' \left(\frac{d\sigma}{d\Omega_{e'} d\omega'} \right)_{IA} f_{\mathbf{q}}(\omega - \omega')$$

$$= \sqrt{T_{\mathbf{q}}}\, \delta(\omega - \Delta) + (1 - \sqrt{T_{\mathbf{q}}})\, F_{\mathbf{q}}(\omega - \Delta)$$

where $T_{\mathbf{q}}$ is the nuclear transparency measured in (e, e', p)

The folding function

- ⋆ The folding function is obtained from the NN scattering amplitude

$$A_{|\mathbf{q}|}(k) = \frac{|\mathbf{q}|}{4\pi}\sigma(i+\alpha)\mathrm{e}^{-\beta k^2}$$

usually parametrized in terms of total cross section, slope and ratio between the real and the imaginary part

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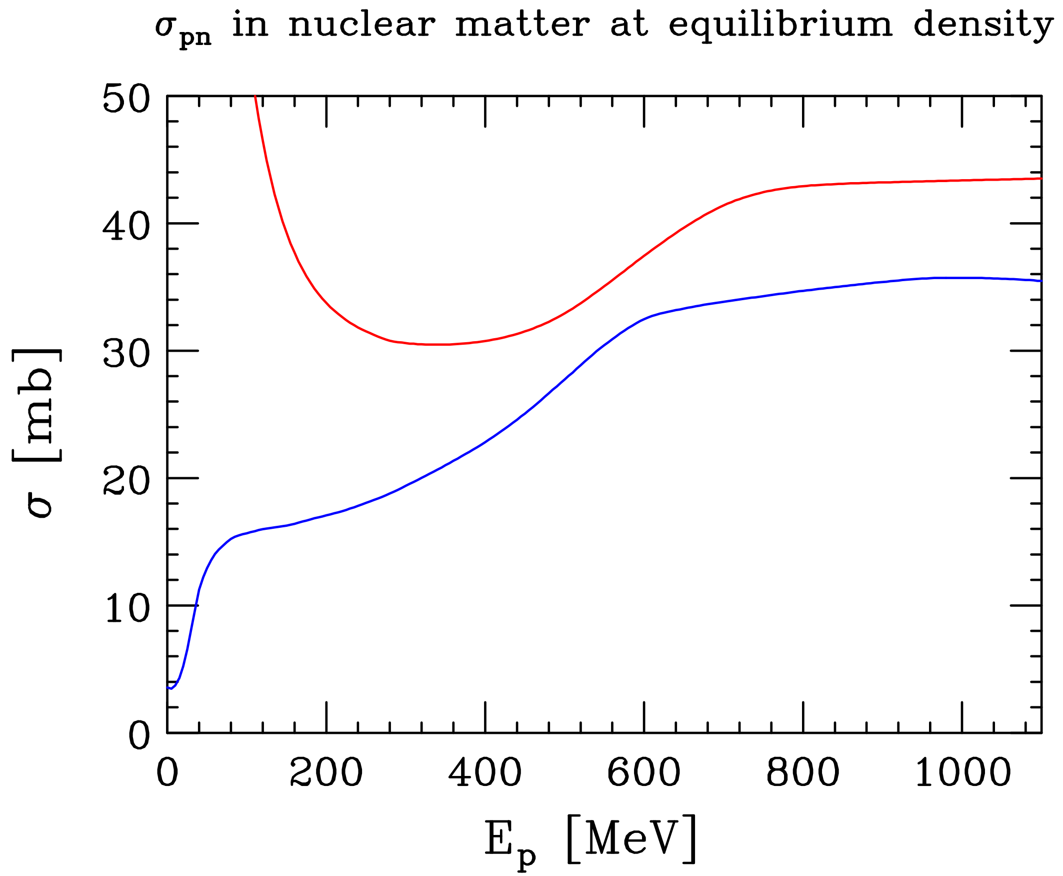
- ⋆ Warning: the total NN x-section σ measured in free space must be corrected to account for the presence of the nuclear medium
- ⋆ NN correlations affect the distribution in space of the spectators particles, strongly suppressing the probability of FSI within ~ 1 fm of the em interaction vertex.

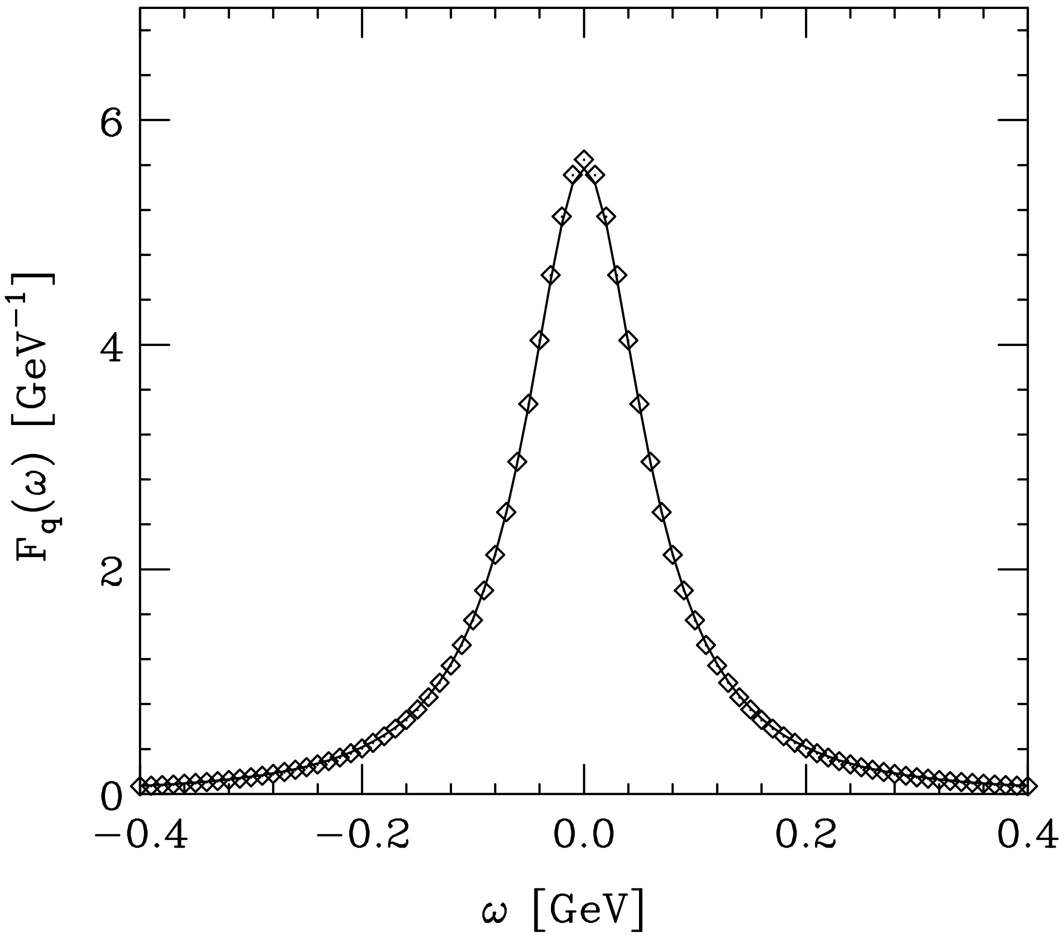
Medium modified NN cross-section

⋆ results of NMBT calculation including Pauli blocking and dispersive corrections



Folding function describing the effect of FSI

⋆ Nuclear matter at equilibrium density, $|\mathbf{q}| = 2$ GeV and 3 GeV



Cross section ratio

- Ratio
$$R = \frac{2}{56}\frac{d\sigma(e + {}^{56}Fe \to e' + X)}{d\sigma(e + {}^{2}H \to e' + X)}$$
at $E_e = 3.6$ GeV and $\theta_e = 25^\circ$ (SLAC data)

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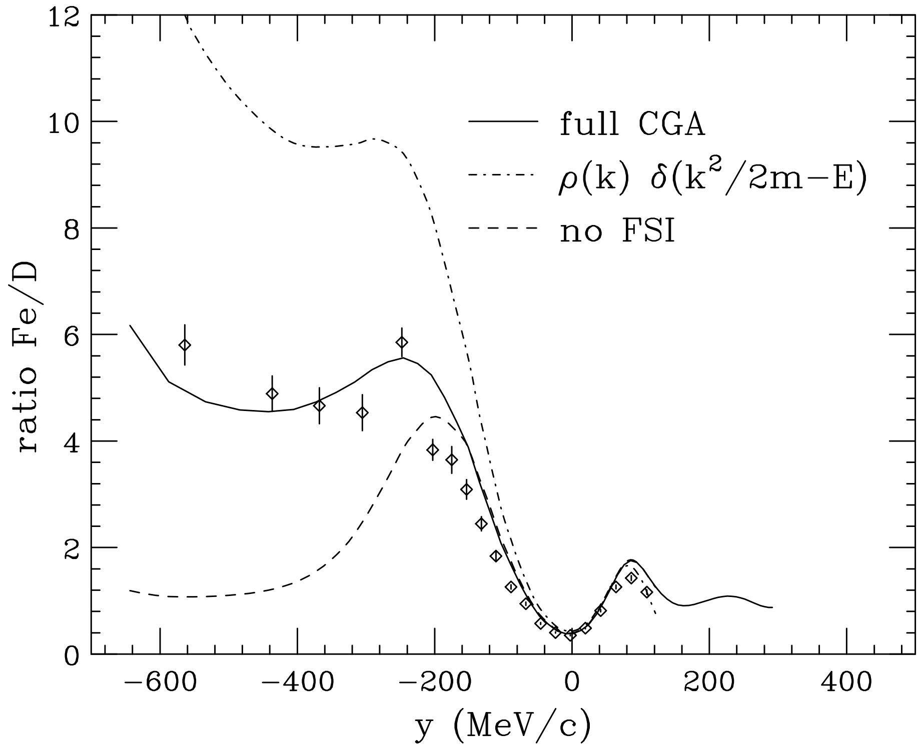
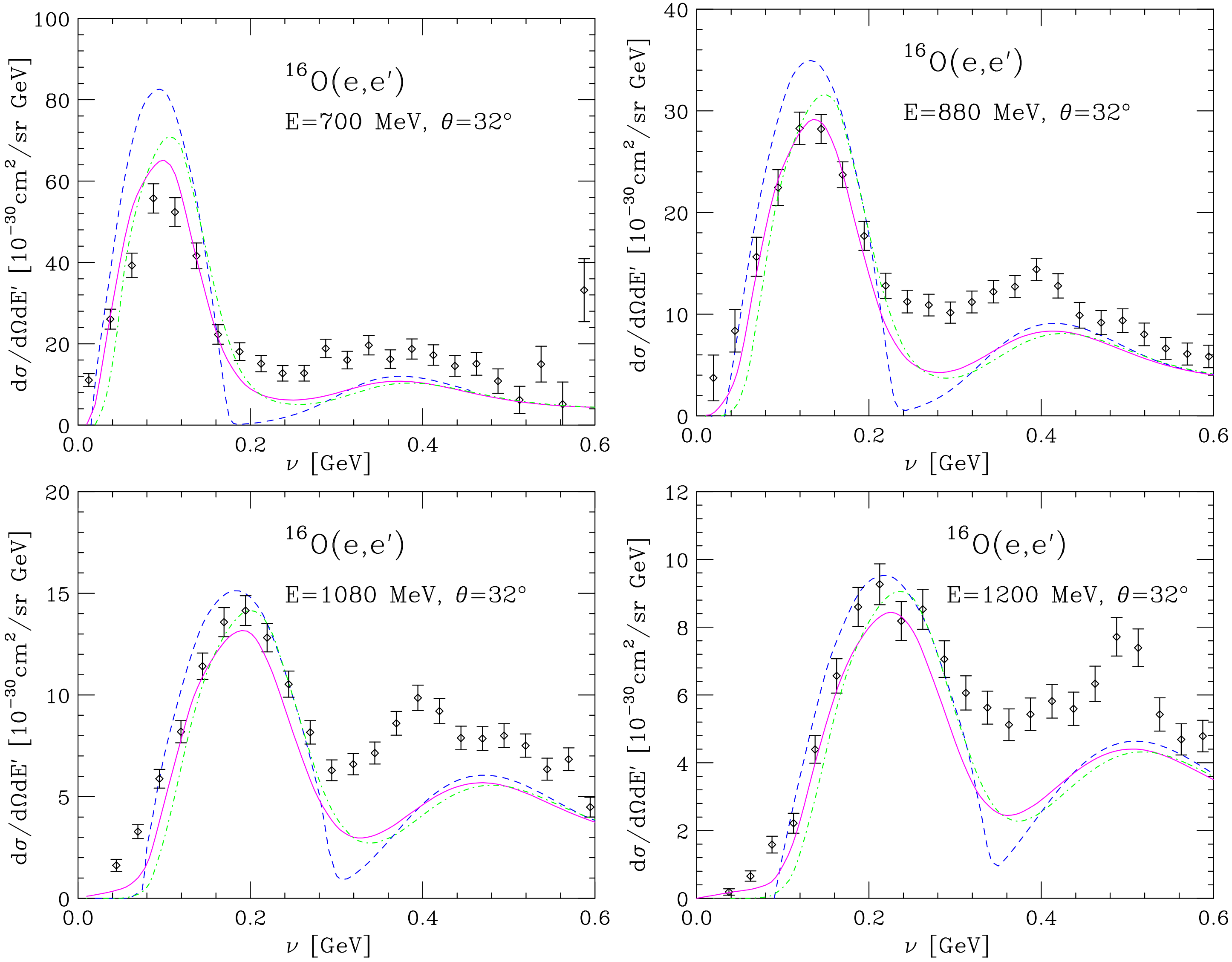
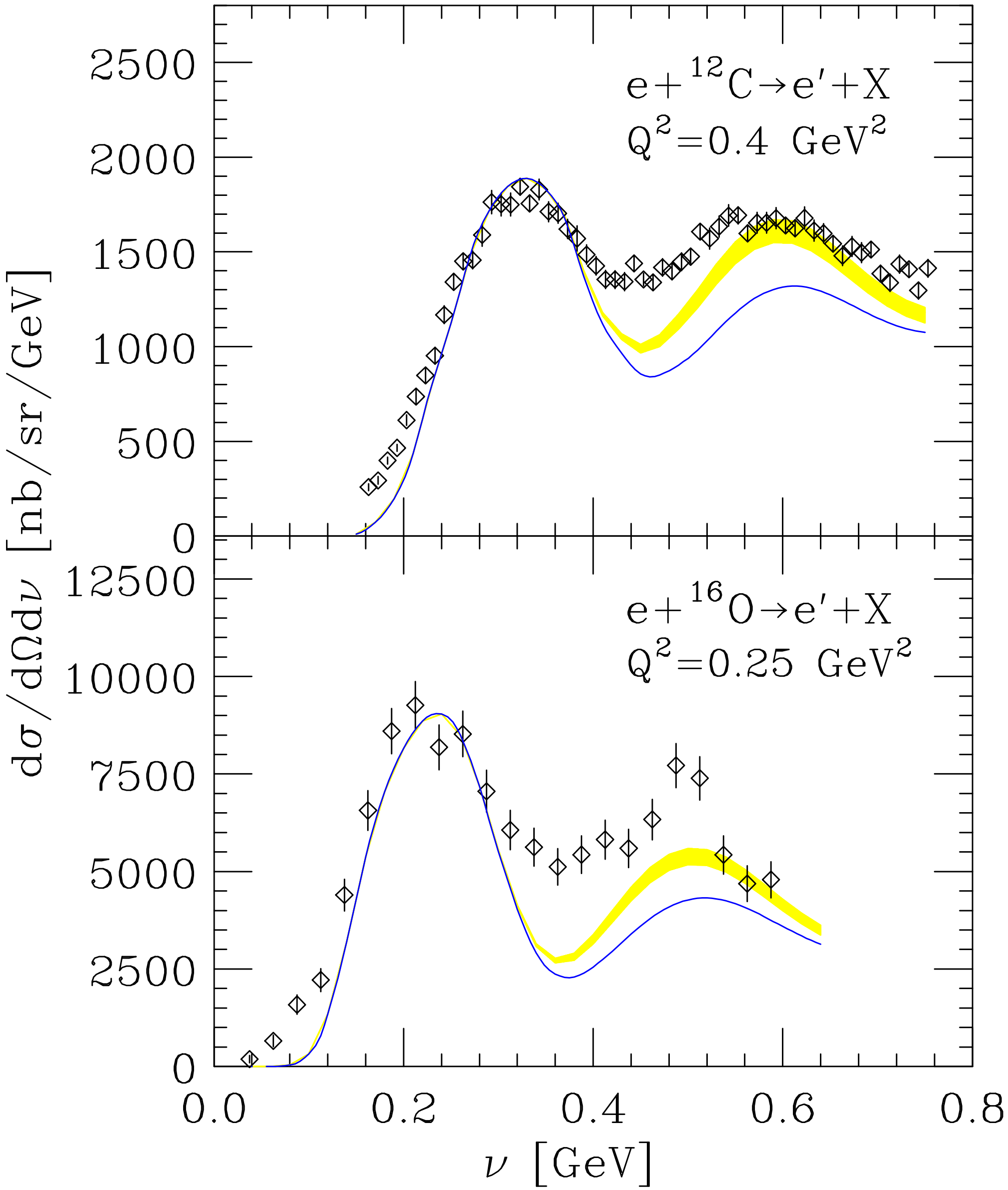


Comparison to Oxygen data @ $0.2 \lesssim Q^2 \lesssim 0.6$ GeV^2

(LNF data, Anghinolfi et al)



Improved description of the Δ resonance region (fit to JLab data)



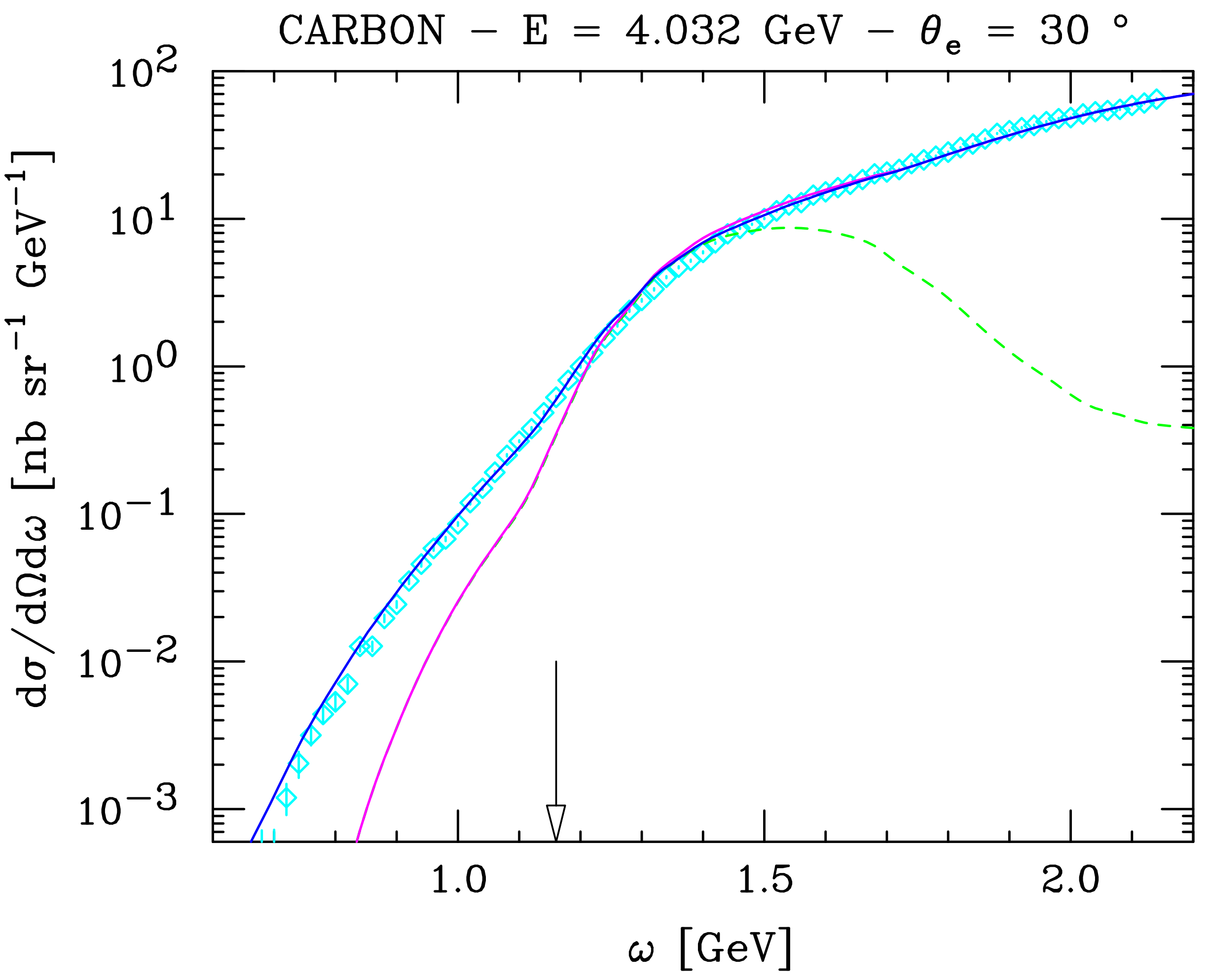
- SLAC data

  (Sealock *et al* (1989))

- LNF data

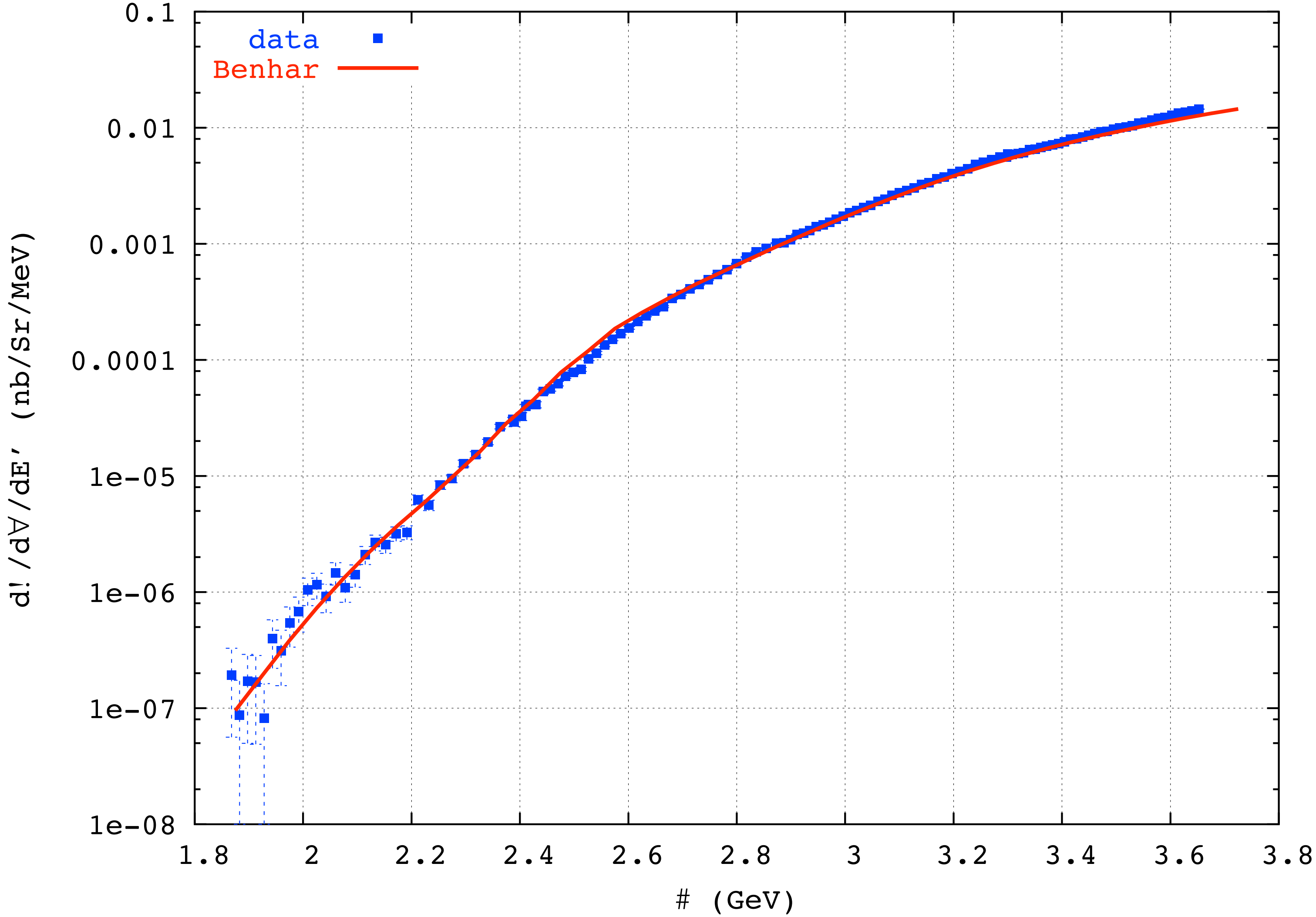
  (Anghinolfi *et* al (1996))

⋆ Comparison to JLab E89-008 data

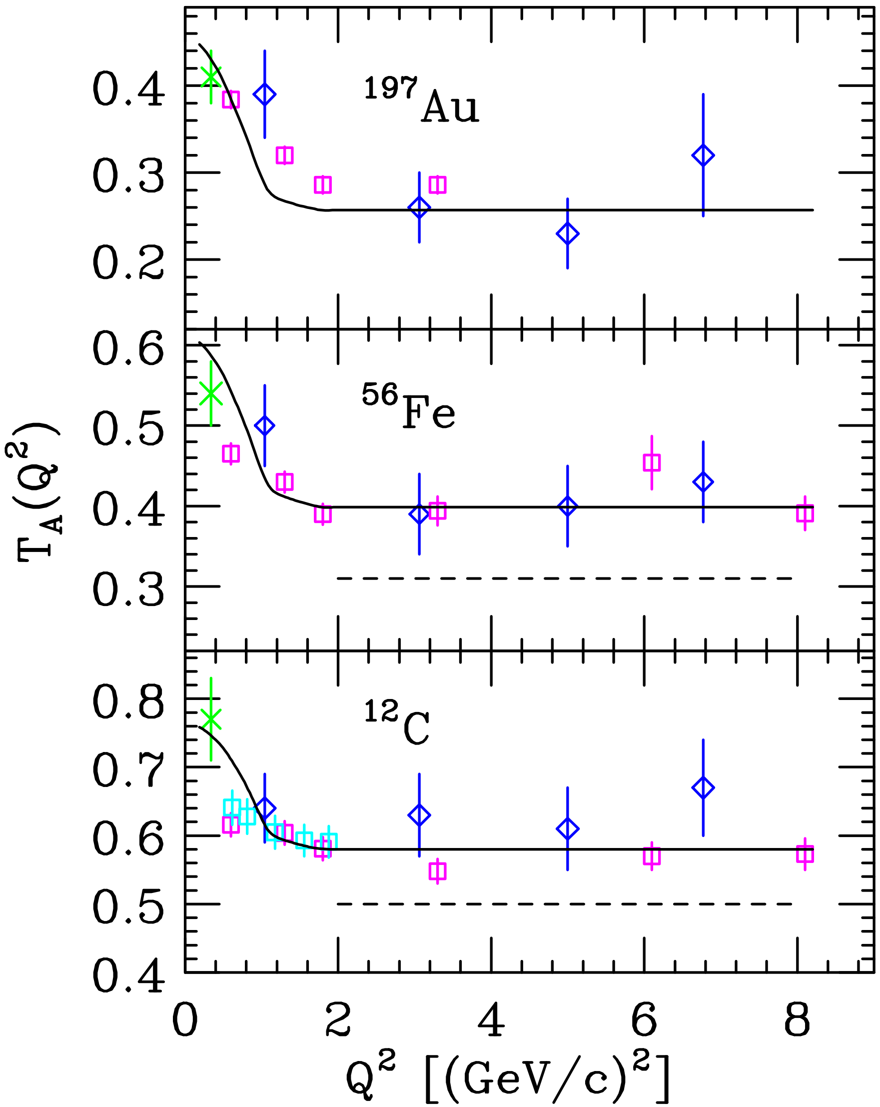


⋆ Note: the arrow points to the kinematical limit of the FG model

⋆ Comparison to JLab E02-019 data: Carbon target, $E = 5.8$ GeV, $\theta_e = 32°$



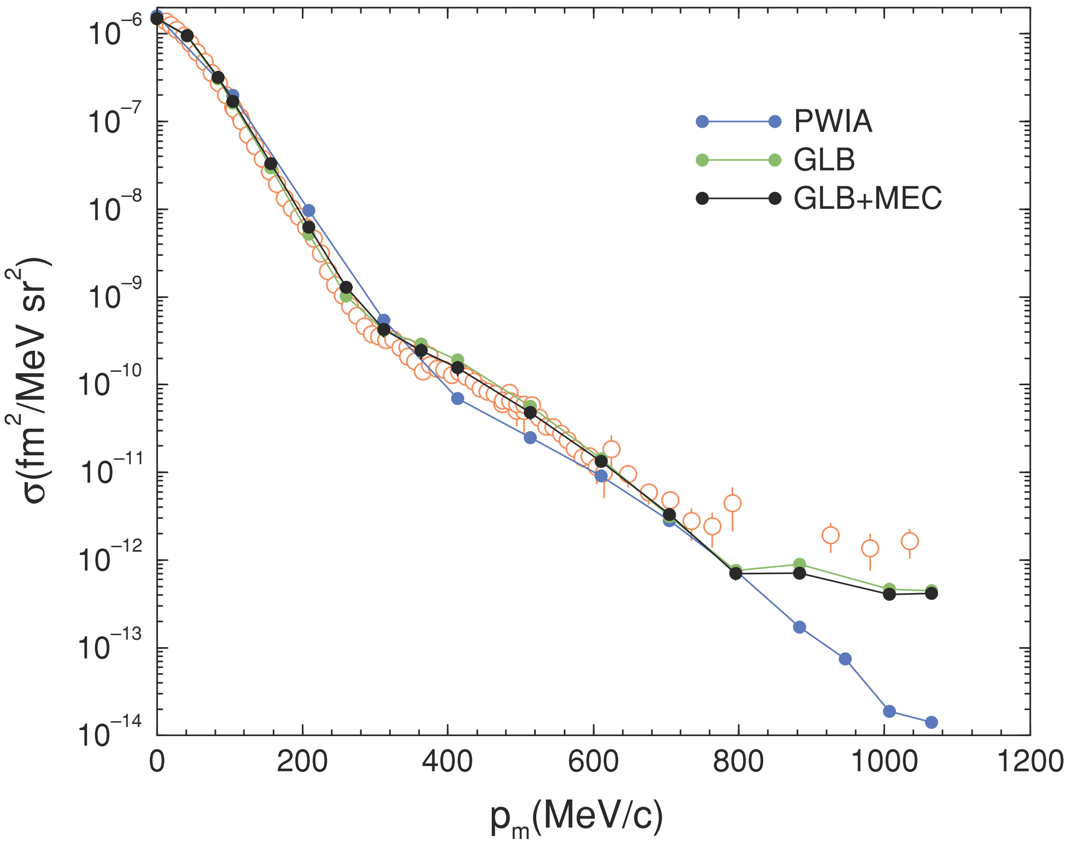
Applications to $(e, e'p)$



- ⋆ Note: in the absence of FSI $T_A = 1$
- ⋆ Data from MIT-Bates, SLAC and JLab

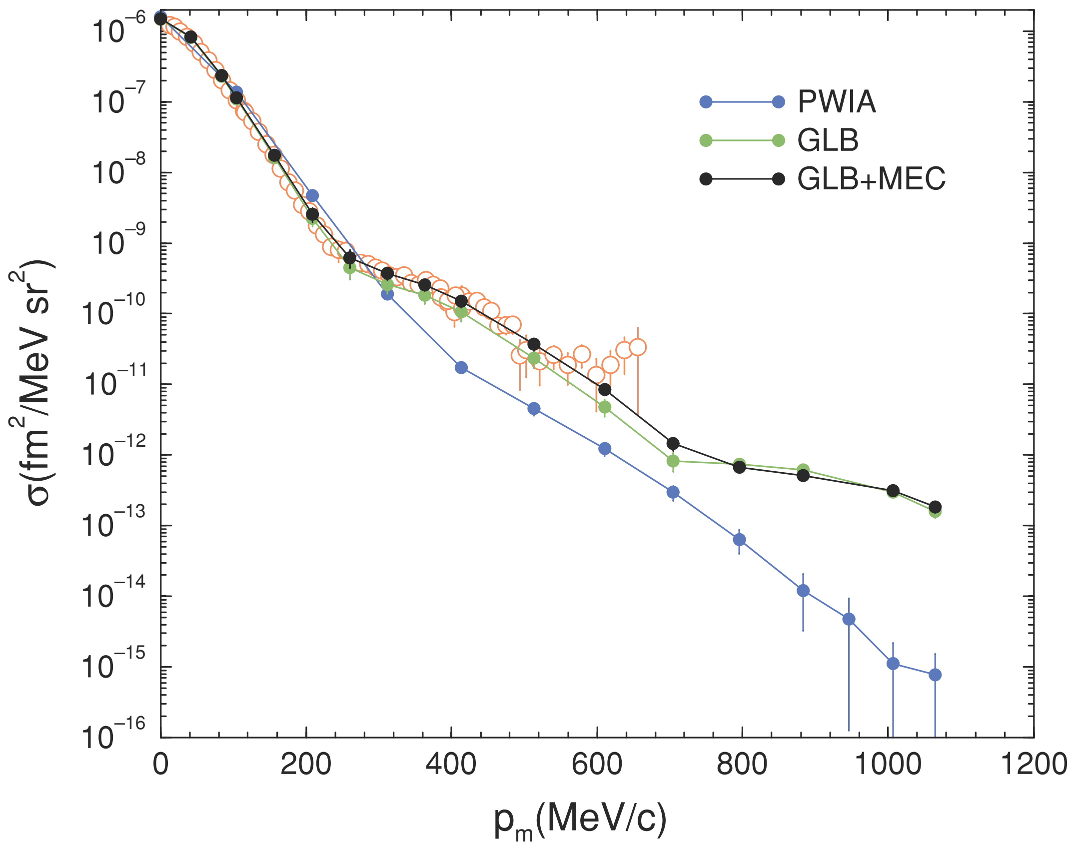
$^3He(e,e'p)d$

- ⋆ Data: Jlab E89044



$^3He(e,e'p)d$ (continued)

- ⋆ Data: Jlab E89044



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  - calculations involving no adjustable parameters (spectral functions)
  - electron-proton and electron-deuteron data (elementary interaction vertex)
- Comparison between theoretical results and electron scattering data shows a remarkably good agreement for a variety of observables in a broad kinematical domain